Todypoon in GIA vol XIII

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THREE-MEMBER COMMITTEE LOOKING FOR A SPECIALIST WITH TWO HIGH ABILITIES

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July 7, 2007

ABSTRACT. An optimal stopping problem belonging to 3-player 2-choice n-stage sequential game is studied. A three-player committee is looking for a specialist with two kinds of high abilities. There are n applicants. The committee interviews each applicant one-by-one and it decides it's R(=reject) or A(=accept) by simple majority over each member's choice of R/A. Payoff to each player is the minimum of the two kinds of abilities of the applicant accepted by the committee. The solution without using computer is given.

1 Statement and Formulation of the Problem. A 3-player (=member) committee has players I, II and III. The committee wants to employ one specialist among n applicants. It interviews applicants sequentially one-by-one. Facing each applicant player I(II, III) evaluates the management ability at $X_1(Y_1, Z_1)$ and computer ability at $X_2(Y_2, Z_2)$. Evaluation by the players are made independently and each player chooses, based on his evaluation, either one of R and A. The committee's choice is made by simple majority. If the committee rejects the first n-1 applicants, then it should accept the n-th applicant. Denote

(1.1)
$$\xi = x_1 \wedge x_2, \eta = y_1 \wedge y_2, \zeta = z_1 \wedge z_2.$$

If the committee accepts an applicant with talents evaluated at x, y, z by I, II, III, resp., then the game stops and each player is paid ξ, η, ζ to I, II, III, resp.. If the committee rejects an applicant, then the next applicant is interviewed and the game continues. Each player of the committee aims to maximize the expected payoff he can get.

The two different kinds of talents (management and computer abilities) for each applicant, are bivariate r.v.s, i.i.d. with pdf

$$(1.2) h(x_1, x_2) = 1 + \gamma(1 - 2x_1)(1 - 2x_2), \ \forall (x_1, x_2) \in [0, 1]^2, |\gamma| \le 1$$

for player I. For II and III, pdfs are $h(y_1, y_2)$ and $h(z_1, z_2)$ respectively, with the same γ . If $X_1(X_2)$ for I is the evaluation of ability of management (foreign language), then γ will be $0 \le \gamma \le 1$. If X_2 is the evaluation of the computer ability, then γ may be $-1 \le \gamma \le 0$.

The bivariate pdf (1.2) is one of the simplest pdf that has the identical uniform marginal and correlated component variables. The correlation coefficient is equal to $\gamma/3$.

Denote the state $(j, \mathbf{x}, \mathbf{y}, \mathbf{z})$ where $\mathbf{x} = (x_1, x_2)$, etc., to mean that (1) the first j - 1applicants were rejected by the committee, (2) the j-th applicant is currently evaluated at x, y, z, by I, II, III resp. and (3) <math>n-j applicants remain un-interviewed if the j-th is rejected by the committee. The state is illustrated by Figure 1.

²⁰⁰⁰ Mathematics Subject Classification. 60G40, 90C39, 90D10.

Key words and phrases. Optimal stopping game, three-player two-choice sequential game, simple maiority rule.