A Hybrid Approach using Snakes and Fuzzy Modelling for Off-Line Signature Verification

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Abstract
This paper introduces an improved snake algorithm based on the Kass et al. proposal. Our approach is applied to the off-line signature verification problem. In off-line verification methods, signatures are scanned and then converted into binary images. This way no dynamic information of the signers is available. Our system uses only one training signature per subject. We also have considered practical conditions for the verification problem when applied to bank checks. Involved system parameters are tuned to solve the task in an effective and efficient manner. Finally, a two-layer perceptron is constructed for signature classification and it uses only two signature features (distance and matching factors, respectively) provided by our snake algorithm. A study of the global system biometric errors for a signature database is also provided.

Keywords: off-line, signature, verification, snakes, fuzzy.

1. Introduction
Signatures are a special case of handwriting subject to intra-personal variations and interpersonal differences. As stated by Justino et al [10]: “signature verification problem aims to maximize the interpersonal differences and to minimize the intrapersonal differences”. This variability makes necessary to analyse signatures as complete images and not as collection of letters and words. Human signatures provide secure means for authentication and authorisation in legal and banking documents; therefore the need of research in efficient automatic solutions for the involved signature recognition and verification problems has increased [15]. In the signature recognition (or identification) problem, a given signature is searched in the database to establish the signer's identity. Signature verification problem is concerned to determine if a particular signature is genuine or is a forgery. Techniques for solving both the recognition and verification problems can be classified as on-line and off-line. In the first ones, data are obtained using an electronic tablet and other devices and in the second ones, images of signatures written on a paper are scanned and dynamic information is not available.

Many approaches for the automatic off-line verification problem have been reported in the literature [4][6][9][12]. In general, the proposed techniques use either a type of features (global, local, statistical, geometric, etc) or a combination of different types of features, extracted from the signature images. In general, this problem has been considered by many authors under controlled conditions with promising results. However, we are far away from an automatic verification system which performs this task under practical conditions with the same effectiveness of a human minimally trained for solving the considered problem.

The paper is organised as follows. Section 2 illustrates the practical off-line signature verification problems. Section 3 describes the signature database used in the experiments. In section 4, we give an overview of the snakes applied to signature images. Section 5 describes the proposed snake-based algorithm for automatic off-line signature verification. Section 6 describes the experimental results produced by a fuzzy model which uses the features extracted by the snake method. Finally the last Section outlines the conclusions and describes the future work.

2. Signature verification problems
Real problems involved in off-line signature verification can be classified in two main categories: (a) those related to the extraction of the signature from the document and (b) those
derived from the characteristics of the verification task. The first group includes problems originated by the need of segmenting the signature from the image document. In general, there is a lack of knowledge about the exact position of the signature in the document. Other related problems are the presence of Gaussian noise caused by document scanning, the existence of texture and logotypes in the document background (i.e. in bank cheques), and the possibility of stamp superposition and typed text mixed with the signature, among others. Fig. 1 shows some examples of situations where the signature segmentation becomes a difficult task.

![Figure 1](image)

**Figure 1.** Some difficult-to-segment signature images, presenting: (a) noised background with typed text, (b) textured background.

Among the problems related to the own verification process, some authors have remarked the lack of sufficient signature samples from each writer for training up the system [13][14]. This difficulty is a very realistic problem due to the private nature of signatures. It would also be desirable to collect signatures from a writer in different periods of time to capture the intrapersonal signature variations. Moreover, with an insufficient number of training samples, the characterization of interpersonal signature differences among writers could result unreliable. Other related signature verification problems are the scalability of the system when adding new writers, the acceptable response time of the system when automatically processing a large amount of documents, and how to consider FAR (False Acceptance Rate) and FRR (False Rejection Rate) values depending on the particular application of the verification system.

It is also very important how to work with the presence of forgeries. These can be roughly classified in two groups: skilled and simple forgeries [12]. The first group corresponds to the falsifiers who know the subject’s signature and can reproduce this signature with a high quality. The second group corresponds to those falsifiers who have never seen the subject’s signature and therefore they can only reproduce with a very low degree of fidelity with respect to the original signature. It is interesting to observe that about 95% of related bank fraud corresponds to simple forgeries [8].

We have analysed and considered all these real practical problems when designing and implementing our off-line signature verification system.

### 2.1. Practical requirements of an off-line signature verification system

A first functional requirement is that each system client can not be required for signing many times. Of course, a reasonable number of signatures per writer would be very desirable for the system training stage. To have a more realistic scenario, we have considered that only one original signature for each system user will be available. However, it is reasonable to consider that this signature is captured under controlled conditions and it would be located in a known position in the document.

Another important requirement is the system scalability with respect to new clients whose signature needs to be verified. Additionally, the system needs to be independent with respect to spatial and radiometric signature resolution conditions.

Finally, high accuracy and fast response time requirements are also demanded by a verification system. Also, the approach needs to be adapted to the particular requirements of each application area. For example, in bank environments is preferable that the system accepts a false signature (FAR error) to the rejection of a true signature (FRR errors), whenever the economic cost of the error is low.

### 2.2. Related works

Automatic signature verification is an active research area since 1975 [12]. As an example, there are many related works concerned with the location of a signature in a noisy environment [18]. Others consider the problem of having only one signature per writer during the system training stage [7], the scalability problem [4], or those problems produced by the skilled forgeries [6], among others.

In general, many existing papers share a standard Image Processing approach [18][9]. This includes a signature pre-processing stage, a segmentation task, a feature extraction stage,
and finally a classification algorithm based on the extracted features.

Some of the most used signature features are: centres of gravity, baselines, upper and lower signature limits, number of holes, signature skeleton, bounding box, signature contour or perimeter, major and minor signature axis, area-to-perimeter ratio, density of points the different signature regions, slant angle, number of signature strokes, crossing points, and so on [19][17].

Among the many referenced off-line signature verification techniques, HMM-based approaches [10][4], fuzzy logic [9], neural networks [15], genetic algorithms [14], elastic graph matching techniques [6] or optimal displacements functions [7] can mentioned.

3. Signature database

Due to the private nature of a signature, there is a lack of standard databases referenced in the literature [12][19]. For this reason, we have created our own signature database that can be obtained in the following URL: http://gavab.escet.urjc.es.

Figure 2. Four signature samples from two different subjects.

This database has actually the signatures of 56 individuals with 6 samples per writer. These patterns were acquired in different periods of time and also using different types of pens. Later, signatures were scanned as binary images with a spatial resolution of 300 dpi and stored as BMP files. Fig. 2 shows some samples of our signature database.

4. Snake overview

A Snake [11][5] is a kind of Active Contour Model [7][3], based on the analysis of the movement of a closed or open contour over an image to which it tries to adjust. An associated energy function, with both internal and external constraint components, is associated for the snake. The internal energy is due to some restrictions of elasticity and flexibility imposed to the snake. The external energy component is caused by the influence of the image which guides the snake movements. The aim is to minimize the energy of the snake which is attracted by the image features such as time or edges.

A parametric representation is generally used to describe the snake position on a two-dimensional image. In this way, this position can be represented as a list of points: \(v(s) = (x(s), y(s))\), and the snake energy as:

\[
E_{\text{snake}} = \int_0^1 E(v(s))ds = \int_0^1 \left[ E_{\text{int}}(v(s)) + E_{\text{image}}(v(s)) + E_{\text{cons}}(v(s)) \right]ds \tag{1}
\]

where \(s\) represents a positional parameter in the snake, \(E_{\text{int}}\) represents the energy due to the relationships among the snake points, \(E_{\text{image}}\) represents an energy related to the positions of the snake points on the image, and finally \(E_{\text{cons}}\) considers the energy associated to other external conditions.

4.1. Internal energy

For many authors [11], the term \(E_{\text{cons}}\) is not considered in eq. (1), and for \(E_{\text{int}}\) the following equation is proposed:

\[
E_{\text{int}} = \frac{1}{2} \left( \alpha(s) \left[ x^2_s(s) + y^2_s(s) \right] + \beta(s) \left[ x^2_{ss}(s) + y^2_{ss}(s) \right] \right) \tag{2}
\]

In (2), \(x_s\) and \(y_s\) represent the respective first derivatives with respect to the positional parameter \(s\). Similarly, \(x_{ss}\) and \(y_{ss}\) are the respective second derivatives. Coefficients \(\alpha(s)\) and \(\beta(s)\) respectively correspond to the influence of the snake elasticity and flexibility. High values of \(\alpha(s)\) produce the snake shape shrinks. High values of \(\beta(s)\) produce that the snake shape tends to be smoother.

Different snake formulations have been proposed to solve related snake problems under particular conditions [5]. In general, all the models are based on spring-like equations.

4.2. Image energy

An energy function is defined for the image where the snake is placed and it moves. This function assigns a value to each image pixel in such a way that the lowest energy values are assigned to the desired final position of the snake. The gradient image is obtained to give the lowest energy values to those pixels with
highest gradient in order to promote the *snake* displacements in a gradient descent direction.

The original *snake* formulation by Kass et al [11] considers different terms that respectively attract the active contour model to the image lines, edges and termination pixels. These elements must be previously segmented. Thus, the image energy $E_{\text{image}}$ was formulated as:

$$E_{\text{image}} = E_{\text{lines}} + E_{\text{edges}} + E_{\text{term}}$$

The first term of the previous equation is proportional to the own image function $I(x,y)$:

$$E_{\text{lines}} = w I(x,y)$$

To attract the snake towards image edges, the following gradient function is proposed:

$$E_{\text{edges}} = -\nabla I(x,y)$$

Finally, to attract the *snake* towards the ending image points, a curvature function is defined on the smoothed image $C$ that is obtained using a Gaussian filter:

$$E_{\text{term}} = \frac{\partial \theta}{\partial n}$$

where: $\theta = \arctan(Cy/Cx)$ and $n = (-\sin(\theta), \cos(\theta))$

### 4.3. Minimizing the *snake* energy

When searching a minimum of the *snake* energy, several assumptions are established to simplify the problem. First, the contour is discretized and it is represented by a sequence of $n$ control points $\{v_i\}$ that define a *spline* or even a simple polygon. Second, the coefficients of the internal energies, $\alpha(s)$ and $\beta(s)$, are converted into constants. Third, the equation (1) is transformed into an iterative formulation. This allows the computation of the successive *snake* positions in time control points $\{v_i(t)\}$, thus obtaining a variational problem.

Amini et al. [1] have proposed a dynamic programming to avoid the repeated and more expensive evaluation of the multiple positions associated to each of the *snake* movements. We have used a modification of this approach in our implementation. According to Amini et al. formulation, the energy defined by the equation (1) can be decomposed into successive stages according to:

$$E(v_1, v_2, \ldots, v_n) = E_1(v_1, v_2) + E_2(v_2, v_3) + \ldots + E_n(v_{n-1}, v_n)$$

Also, the computation of each energy term can also be decomposed, using dynamic programming scope, into a sequence of functions of a variable $s$, where the set $\{v_i\}$ is obtained by the following system of recurrence equations:

$$s_i(v_j) = \min_{v_k} \{E_i(v_j, v_k) + s_{i+1}(v_k)\}$$

$$E_1(v_1, v_2) = w I(v_1, v_2)$$

To evaluate the complete equation (2), we have to consider that computing the second derivative involves a third point. The energy term can be rewritten as follows:

$$E(v_1, v_2, \ldots, v_n) = \sum_{i=1}^{n-1} \left( E_i(v_{i-1}, v_i, v_{i+1}) \right)$$

where:

$$E_i(v_{i-1}, v_i, v_{i+1}) = E_{\text{img}}(v_i) + E_{\text{int}}(v_{i-1}, v_i, v_{i+1})$$

In consequence, the set $\{v_i\}$ will be expressed as:

$$s_i(v_{i-1}, v_i, v_{i+1}, v_{i+2}) = \min \left\{ s_{i-1}(v_{i-1}, v_i) + \alpha |v_i - v_{i-1}|^2 + \beta |v_{i-1} - 2v_i - v_{i+1}|^2 + E_{\text{int}}(v_i) \right\}$$

### 5. Proposed method using *snakes*

The signature verification problem consists in minimizing the adjustment energy applied to the *snake* (obtained from a model signature) and the image of the test signature.

The proposed verification algorithm can be resumed in the following steps:

1) For each known signature, create a polygonal line (*snake*) $P$, composed by equal-spaced control points, using a model signature image.

2) Place approximately $P$ over a signature to verify.

3) Use the considered *snake* algorithm to adjust $P$, as best as possible, to the new image.

4) Compare a measure of the energy necessary to deform the *snake* after the adjustment process. This value is compared with a threshold and is used to decide whether or not the test signature is genuine or is a forgery.

#### 5.1. Initial adjustment method

As a first approximation, the original *snake* definition by Kass et al [11] was used. As the signature images were binary, finding the edges and ending image points makes no sense to
adjust the *snake*. Only the energy associated to each image pixel represented by eq. (3) was considered to attract the *snake* towards the signature image. For simple images, the results were promising since the *snake* adjusted accurately in a few number of iterations to the signature image. However, this model has two main problems: the influence of the position where the *snake* is initially located, and the significant lost of the original shape of the *snake*.

\[ E_{\text{snake}} = E_{\text{image}} + E_{\text{shape}} \]

Potential maps

Cohen and Cohen [16] have proposed the use of an external force, based on the idea of potential map, defined as the Euclidean distance in pixels \( m_{\text{image}}(x,y) \) from each image no signature point to the nearest image signature point. In this way, the algorithm first computes the distance values beginning from each of the adjacent points to the signature strokes and then moving away. Fig. 4 shows the potential map corresponding to an example signature. The intensity level associated to each image point reflects the distance to the signature pixels.

\[ E_{\text{image}}(v_i) = m_{\text{image}}(v_i) \]

5.2. Improved adjustment method

As a second approximation, a new *snake* energy definition \( E'_{\text{snake}} \) was adapted with two considered aspects. First, to solve the snake locality problem, a potential map for the energy term \( E_{\text{image}} \) attached to each snake control point \( v_i \) is defined as:

\[ E'_{\text{image}}(v_i) = m_{\text{image}}(v_i) + m_{\text{image}}(v_{i-1}) + m_{\text{image}}(v_{i+1}) \]

where \( i \) is defined in the range 1..\( n \).

Shape conservation

To avoid an excessive *snake* deformation during the iterations, the internal energy of the snake is characterized as:

\[ E'_{\text{shape}} = E'_{\text{angle}} + E'_{\text{prop}} \]

The term \( E'_{\text{angle}} \) serves to maintain the angle between each pair of adjacent *snake* segments into a controlled interval. This can be expressed as:

\[ E'_{\text{angle}}(v_{i-1}, v_i, v_{i+1}, t) = \left\{ \begin{array}{ll} \infty & \text{if } \delta > U_a \\ k_a \cdot \delta & \text{if } \delta \leq U_a \end{array} \right. \]

where:

\[ \delta = \left| \text{ang}(v_{i-1}(t), v_i(t), v_{i+1}(t)) - \text{ang}(v_{i-1}(0), v_i(0), v_{i+1}(0)) \right| \]

and \( \text{ang} \) represents the angle defined by the three vertices: \( v_{i-1}, v_i \) and \( v_{i+1} \), and \( k_a \) is a constant.
The term $E'_{prop}$ is introduced to preserve the proportions between adjacent segments in $P$. The ratio $\phi$ is defined by the quotient between a pair of adjacent segments in the \textit{snake} after $t$ iterations and the same pair of the adjacents segments before the iterations. The function defined by $E'_{prop}$ can be expressed as:

$$E'_{prop}(v_{i-1},v_i,v_{i+1},t) = \begin{cases} 
\infty & \text{if } 1 - U_p \cdot \phi > 1 + U_p \\
\phi & \text{if } 1 - U_p \leq \phi \leq 1 + U_p 
\end{cases}$$

where $\phi = \frac{|v_i(t)v_{i+1}(t)|}{|v_i(0)v_{i+1}(0)|}$

Equations corresponding to $E'_{angle}$ and $E'_{prop}$ are compatible with the energy minimization formulation given by equation (4).

5.3. Similarity measure

After a variable number of iterations, the \textit{snake} converges to a solution. To study the similarity between the \textit{snake} and the signature to which it is adjusted, other elastic matching algorithms were tested [17][3], achieving poorer results.

We now introduce two discriminant features which are used for signature classification: the coincidence measure and the distance measure.

\textbf{Coincidence measure}

This factor $f_c$ uses a potential map $m_{image}(x,y)$ to assign a value to each snake point (not only the control points). In order to obtain a normalized value between 0 and 1, the measure $f_c$ is computed as follows:

$$f_c = 1 - \frac{1}{N} \sum_{i=0}^{N} c(p(i))$$

where: $c(p(i)) = \frac{m_{image}(p(i))}{k_f \cdot g}$

and $N$ represents the number of \textit{snake} points, $p(i)$ are all the points belonging to the \textit{snake} segments, $g$ is the average thickness in pixels of the signature strokes and $k_f$ is a scaling factor.

\textbf{Distance measure}

This factor $f_d$ also uses a potential map $m_{snake}(x,y)$, which permits to compute the distance of the signature pixels with respect to the \textit{snake} position. The distance measure is computed as:

$$f_d = \sum_{j=0}^{M} d(r(i))$$

where: $d(p(i)) = \begin{cases} 1 & \text{if } m_{snake}(r(i)) < g \\
0 & \text{if } m_{snake}(r(i)) \geq g 
\end{cases}$

and $r(i)$ are the signature pixels, $M$ are the number of these pixels and $g$ is the average thickness in pixels of the signature strokes. This second measure can be influenced by the structural noise.

6. Fuzzy Modelling

To establish the degree of genuineness of a given test signature, a fuzzy model is proposed. Due to the shape and size variability in signatures of the same subject, it is even difficult for humans to distinguish between an authentic signature and a forgery. Moreover, the available information contained in a signature, that is captured off-line, may be in general imprecise and difficult to extract. Consequently, a system with tolerance to imprecision can be developed to achieve a more robust, tractable and low-cost solution [20]. These comments suggest the possibility of using a fuzzy modelling technique for the considered automatic signature verification problem.

In our particular application, we go in for fuzzy modelling of the two extracted features, respectively, coincidence measure ($f_c$) and distance measure ($f_d$). For the purpose of signature verification (and detection of forgeries), we have used a first-order Takagi-Sugeno (TS) model. Hanmandlu et al [2] have used a similar approach but considering angle distribution of signature pixels in regions as features. The two distributions of $f_c$ and $f_d$ values for the genuine signatures (see Fig. 5), define two fuzzy sets $A_c$ y $A_d$. Let $x_k$ the $k^{th}$ ($k=1...2$) feature in the fuzzy set $A_k$, so the $k^{th}$ IF-THEN...
fuzzy rule in the first-order TS model, can be written as:

Rulek : IF \( x_k \) is \( A_k \) THEN \( y_k = c_k+d_kx_k \) \( k=1,2 \)

The fuzzy set \( A_k \) can be represented by the following Gaussian membership function which includes the parameter \( a_k \):

\[
\mu_k(x_k) = e^{-\frac{(x_k-a_k)^2}{b_k}} \quad k=1,2
\]

The output of the fuzzy system is a crisp value obtained as follows:

\[
o = \sum_{i=1}^{2} \mu_i y_i
\]

This value \( o \) defines the resulting similarity of the test signature with the model signature to which it is compared.

```c
double e[]={0.001, 0.001, 0.05};
double sum_delta;
double x[3][TRAIN_SIZE], nu[2];
double a[2], b[2], c[2], d[2];
double a_n[2], b_n[2], c_n[2], d_n[2];
do{
    sum_delta = 0.0;
    for (int pat = 0; pat < TRAIN_SIZE; pat++){
        for (int i = 0; i < 2; i++)
            nu[i] = exp(-((x[i][pat]-a[i])* (x[i][pat]-a[i])/b[i]) / (b[i]*b[i]));
        double Out = nu[0]*c[0]+d[0]*x[0][pat]+
                    nu[1]*c[1]+d[1]*x[1][pat];
        double delta = x[2][pat] - Out;
        for (int i = 0; i < 2; i++){
            double Out_i = c[i]+d[i]*x[i][pat];
            double d_J_a = -4*delta*nu[i]*Out_i*
                            (x[i][pat]-a[i])/(b[i]*b[i]);
            double d_J_b = -4*delta*nu[i]*Out_i*
                            (x[i][pat]-a[i])*(x[i][pat]-a[i])/
                            (b[i]*b[i]*b[i]);
            double d_J_c = -2*delta*nu[i]*x[i][pat];
            double d_J_d = -2*delta*nu[i]*x[i][pat];
            a_n[i]=a[i]-e[0]*d_J_a;
            b_n[i]=b[i]-e[1]*d_J_b;
            c_n[i]=c[i]-e[2]*d_J_c;
            d_n[i]=d[i]-e[2]*d_J_d;
        }
        for (int i = 0; i < 2; i++)
            a[i]=a_n[i]; b[i]=b_n[i];
            c[i]=c_n[i]; d[i]=d_n[i];
        for (int i = 0; i < 2; i++)
            nu[i] = exp(-((x[i][pat]-a[i])* (x[i][pat]-a[i])/b[i]) / (b[i]*b[i]));
        double Out = nu[0]*c[0]+d[0]*x[0][pat]+
                    nu[1]*c[1]+d[1]*x[1][pat];
        double delta = x[2][pat] - Out;
        sum_delta += delta;
    }
}while (sum_delta*sum_delta > 0.002);
```

Figure 6. C implementation of the parameter tuning algorithm.

The fuzzy model was trained until the error was smaller than a value of 0.01 using the algorithm of Fig. 6. A total of 28 from 56 individuals of the database were used for training the fuzzy model. The other 28 individuals were used for test purposes. For each person \( i \), we have six genuine signatures: one is used to build the snake and the other five to train the fuzzy model. For each person all the signatures of the 27 remaining individuals are used to define the set of patterns to reject by the snake (class) \( i \). By applying this method for the 28 train individuals, we have a set of 28×5=140 train signatures as patterns to accept, and a set of 28×27×6=4536 test signatures as patterns to reject.

Each test signature is compared with the corresponding snake, using the verification method proposed in section 5. After this process, two selected features \( f_c \) and \( f_d \) are computed, and used to test using the trained fuzzy model. The obtained result (real number between 0 and 1) defines the similarity degree with the corresponding model signature. Finally, a threshold value must be chosen in order to decide whether to accept or to reject a test signature. Fig. 7 presents the obtained False Acceptance Rates (FAR) and the False Rejection Rates (FRR) values for different thresholds.

<table>
<thead>
<tr>
<th>Threshold</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>FAR</td>
<td>100</td>
<td>90</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>FRR</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
<td>100</td>
</tr>
</tbody>
</table>

Figure 7. FAR and FRR curves for different threshold values.

<table>
<thead>
<tr>
<th>Signatures</th>
<th>Genuine</th>
<th>Forgery</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>140</td>
<td>4536</td>
</tr>
<tr>
<td>Error</td>
<td>14</td>
<td>422</td>
</tr>
<tr>
<td>Correct</td>
<td>126</td>
<td>4114</td>
</tr>
</tbody>
</table>

Table 1. Results for a threshold equal to 0.53 where FAR equals to FFR.
7. Conclusion

A new snake-based off-line signature verification method is proposed. This algorithm works in practical conditions using only one training signature for each user. The algorithm for adjusting the snake to a handwritten signature is also a result of this paper. Nowadays, we are working in some implementation aspects of the proposed system and in the improvement of the involved algorithms.

In particular, two components are needed to have a complete automatic off-line signature verification system:

(a) A method to automatically define the snake using the training signature image.
(b) An algorithm for the location and the segmentation of the signature in a generic document.

As future work, we propose to improve the comparison method between the iterated snake and the signature being verified. It is also necessary an algorithm parameter optimization in presence of noise.

8. References