A formal approach to Perception Calculus of Zadeh by means of rough mereological logic

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Abstract

Rough set theory is a paradigm for approximate reasoning based on the assumption that concepts are divided into exact and non–exact (called also rough) ones by means of a topological structure induced by a representation of knowledge as a classification. A classification in its most simple form is an equivalence relation on a universe of objects; the classification induces a partition topology and concepts (meant as subsets of the universe) that are clopen are exact whereas other concepts are rough. In consequence, rough sets are represented as pairs of exact sets of the form (interior, closure).

Investigations into deeper structures resulting from that assumption have led to an idea of rough mereology. On this ground also possibility for intensional many–valued logics has been recognized that have been called rough mereological logics.

In this contribution, we present a development of rough mereological logics and we propose an application for these logics as a framework within which Calculus of Perceptions outlined by Zadeh may be given a formal rendering as a tool for semantic interpretation of vague statements.

Keywords: rough sets, rough mereology

1 Introduction: Basic Ideas of Rough Sets

Uncertainty of a concept is captured in rough set theory by the idea of a region of uncertainty around the concept, into which objects may not be classified with certainty but to a degree only. It is worth stressing that the region of uncertainty itself is a crisp set. Rough sets [11] render this idea by formally defining a pair \((U, R)\), where \(U\) is a universe of objects and \(R\) is a relation of equivalence (classification); the pair \((U, R)\) is referred to as a classification (or, an approximation) space.

Equivalence classes \([u]_R\) of the classification \(R\) form elementary granules of knowledge, and unions of them are granules of knowledge called also \(R\)–exact sets; other sets of objects are \(R\)–rough.

For an \(R\)–rough set of objects \(X\), two approximations to \(X\), called respectively, the lower approximation \(\underline{RX}\) and the upper approximation \(\overline{RX}\), are defined as follows,

\[
\underline{RX} = \{u : [u]_R \subseteq X\};
\]
\[
\overline{RX} = \{U : [u]_X \cap X \neq \emptyset\}.
\]

In consequence, a rough set \(X \subseteq U\) is defined as a pair \((\underline{RX}, \overline{RX})\) of exact sets up to the equivalence of having the same lower and upper \(R\)–approximations.

Logics based on rough sets have been studied extensively cf. [6], [7], [8], [9], [16], [17],
[18], revealing modal structures induced by rough set structures, epistemic logics based on knowledge operators induced by rough set–theoretic approximations, and algebraic structures relevant for certain logics. Here, we propose a formalization in terms of intensional logics of a variety of reasoning schemes based on the notion of a part to degree rendered by certain function called rough inclusions, introduced in [15]. We apply those reasoning schemes in a rendering of some aspects of Calculus of Perceptions, a new idea proposed by Zadeh [19], aimed at computing with perceptions, i.e., vague statements about facts of the world expressed often in natural language. We propose an interpretation of perceptions in a twofold way: on one hand, one extracts from perceptions constraints on objects that yield semantically exact sets, on the other hand, one extracts a fuzzy statement that yields a fuzzy predicate whose meaning is a set. Rough mereological logic assigns to this pair the truth value of the fuzzy meaning over the exact constrained set that is interpreted as the truth value of the perception. The paper is concluded with an example that explains the idea.

2 A set theory for rough sets

We begin, in the setting \((U, R)\) of an approximation space, with a new elementship notion, \(\in^*\) by letting,

\[
u \in^* X \iff \langle u \rangle_R \subseteq X,
\]

where \(u\) is any object in the universe \(U\).

Obviously,

**Proposition 1** A concept \(X\) is \(R\)–exact if and only if the dichotomy,

\[
u \in^* X \lor u \in^* U \setminus X,
\]

holds for each \(u \in U\).

We will call the new notion of elementship, the rough elementship notion. We will render this idea in an abstract setting.

We introduce a general notion of the set theory of rough sets, (STRS, for short).

By an instance (a model) of STRS, we mean a tuple,

\[(U, \cup, \cap, \setminus, \in, \in^*, \mathcal{E}, \mathcal{R}),\]

where:

- \(U\) is a set (in the standard ZF theory);
- \(\cup, \cap, \setminus\) are standard set operations of the union, the intersection and the complement (set difference as well);
- \(\in\) denotes the standard elementship predicate \(est\);
- \(\in^*\) denotes a new elementship predicate to be specified below,

and, moreover, the following requirements are satisfied, where the notion of containment \(\subseteq\) is based on the element predicate \(\in\):

1. \(\mathcal{E}\) is a family of subsets of \(U\) such that \(\emptyset, U \in \mathcal{E}\);
2. \((\mathcal{E}, \cup, \cap, \setminus, \emptyset, U)\) is a Boolean algebra;
3. any union of a family of sets in \(\mathcal{E}\) is an element of \(\mathcal{E}\);
4. \(X \in \mathcal{R} \iff X \notin \mathcal{E},\) for each \(X \subseteq U\);
5. \(\mathcal{E}\) is \(\subseteq\)–co–initial in the power set \(2^U\);
6. \(u \in^* X\) if and only if there exists \(Y \in \mathcal{E}\) such that \(u \in Y \subseteq X\).

We now derive some consequences from (1)-(6).

**Proposition 2** For each pair \(x \in U, X \subseteq U\):

- if \(x \in^* X\) then \(x \in X\).

**Proof 1** By (6).

**Proposition 3** The family \(\mathcal{E}\) is \(\subseteq\)–co–final in the power–set \(2^U\).

**Proof 2** By (2) and (5).

**Proposition 4** The Boolean algebra \((\mathcal{E}, \cup, \cap, \setminus, \emptyset, U)\) is \(\subseteq\)–complete.
Proof 3 By (3).

Proposition 5 For each \( X \subseteq U \) there exist sets \( X, \overline{X} \in E \) such that (i) \( X \) is the l.u.b. of the family \( \{Y \in E : Y \subseteq X\} \); (ii) \( \overline{X} \) is the g.l.b. of the family \( \{Y \in E : X \subseteq Y\} \).

Proof 4 By Proposition 4.

Corollary 1 For each \( X \subseteq U, \overline{X} \subseteq X \subseteq \overline{X} \).

Corollary 2 For each \( X \subseteq U, X = \overline{X} \) if and only if \( Y \) is neither \( U \) nor \( \overline{X} \). Assume that \( X = \overline{X} \) if and only if \( X \in E \).

Proof 5 By Proposition 5.

Proposition 6 For each \( X \subseteq U \): the set \( X \in E \) if and only if the dichotomy holds \( x \in^* X \lor x \in^* U \setminus X \) for each \( x \in U \).

Proof 6 Observe that \( x \in^* X \lor x \in^* U \setminus X \) is indeed the dichotomy by Proposition 2 for each pair \( x, X \). Assume that \( X \notin E \) for some \( X \). Then \( x \notin X \) by Corollary 2, so we may pick \( x \in X \setminus \overline{X} \). It follows that if \( x \in Y \in E \) then neither \( Y \subseteq X \) nor \( Y \subseteq U \setminus X \), i.e., neither \( x \in^* X \) nor \( x \in^* U \setminus X \).

Proposition 7 For each pair \( x \in X \in E \) there exists \( Y \in E \) such that (i) \( x \in Y \subseteq X \); (ii) \( Y \) is minimal with respect to the property (i).

Proof 7 By completeness.

We let,

\[
\mathcal{E}_0 = \{Y : Y \in E \text{ such that } Y \text{ is minimal w.r.t. } x, X \text{ and (i) (ii) in Prop. 7}\}.
\]

Let \( \mathcal{E}^* \) be a minimal sub–family of the family \( \mathcal{E}_0 \) with respect to the property that for each \( x \in X \in E \), there exists \( Y \in \mathcal{E}^* \) such that \( x \in Y \subseteq X \).

Proposition 8 Elements of the family \( \mathcal{E}^* \) are pair–wise disjoint.

Proof 8 Assume that \( x \in Y_1 \cap Y_2 \) for some \( Y_1, Y_2 \in \mathcal{E}^* \). Then by minimality of \( Y_1 \), and the fact that \( x \in Y_1 \cap Y_2 \subseteq Y_1 \), it follows that \( Y_1 \subseteq Y_2 \); by symmetry, \( Y_2 \subseteq Y_1 \) and thus \( Y_1 = Y_2 \).

Proposition 9 For each \( X \in E \), \( X \) is l.u.b. of the family \( \{Y \in \mathcal{E}^* : Y \subseteq X\} \).

Proof 9 Obvious by definition of \( \mathcal{E}^* \).

Corollary 3 The family \( \mathcal{E}^* \) is a partition of the set \( U \).

Thus, in a model of STRS, one can reconstruct the partition of the universe into elementary exact sets that generate exact sets. The difference between exact sets and rough sets is expressed by means of the dichotomy stated in Proposition 6 that is satisfied by exact sets exclusively.

A model of STRS is induced naturally in any knowledge base of a more general form of \( (U, \{R_i : i \in I\}) \) by considering classes of the relation \( R = \bigcap_i R_i \), the relation \( \in^* \) defined as \( x \in^* X \) if and only if \( [x]_R \subseteq X \), and \( E \) defined as the collection of \( R \)-exact sets with respect to \( R \).

3 Rough Mereological Logic (RML)

The notion of rough elementship, \( \in^* \), depends on the containment \( \subseteq \), i.e., it is of mereological character, as \( \subseteq \) satisfy the requirements set on the predicate "to be a part of", \( \pi \), in mereology [3], i.e.,

1. if \( x\pi y \land y\pi z \) then \( x\pi z \); 2. \( x\pi x \) for no \( x \). (6)

3.1 A note on mereological theory of sets

Mereology, proposed by Leśniewski (in: On Foundations of Set Theory (in Polish), Polish Scientific Society, Moscow, 1916; for a concise rendition see [3]) based on the predicate of part, is endowed with a class operator whose aim is to convert distributive concepts (collections of objects) into collective concepts (objects).

Starting with the relation \( \pi \), one defines the predicate of an element, \( el_\pi \), as follows,
\( \text{\textit{xel}}_{\pi} y \) if and only if \( x \pi y \vee x = y \). \hspace{1cm} (7)

Then, for a non-empty concept \( M \), the class of \( M \), in symbols \( \text{ClsM} \), is the object that satisfies the properties,

(i) if \( M(x) \) then \( \text{\textit{xel}}_{\pi} M; \)

(ii) if \( \text{\textit{xel}}_{\pi} M \) then \( \text{\textit{ucl}}_{\pi} x, \text{\textit{ucl}}_{\pi} w, \)

\[ M(w) \text{ for some } u, w. \] \hspace{1cm} (8)

The definition (8) of the class \( \text{ClsM} \), renders in the abstract form the mechanism of pasting objects together by means of their common parts in order to form from them a new global objects.

Let us observe that the predicate \( \subseteq \) of proper set containment is a part predicate with the related element predicate \( \subseteq \). For a collection \( M \) of sets, the class \( \text{ClsM} \) with respect to \( \subseteq \) is the union \( \bigcup M \).

### 3.2 Rough mereology

An extension to mereology is rough mereology, first outlined in [15], see [14] that bases itself on the predicate, \( \mu(x, y, r) \), of "being a part to a degree", called a rough inclusion. Here, we propose requirements for \( \mu \) in a slightly modified form, given a mereology induced by the part predicate \( \pi \).

\[
\begin{align*}
1. & \quad \mu(x, x, 1); \\
2. & \quad \mu(x, y, 1) \Rightarrow [\mu(z, x, r) \land \mu(z, y, s) \Rightarrow s \geq r]; \\
3. & \quad \mu(x, y, 1) \Rightarrow \text{\textit{xel}}_{\pi} y.
\end{align*}
\]

(9)

Rough mereology in turn has formed a foundation for such paradigms of soft computing as granular computing [12].

### 3.3 A logic RML

Our procedure for defining RML is as follows (see, e.g., [2], [5] for a discussion and examples of intensional logics). We define an intension \( I \) as a function on the Cartesian product \( \mathcal{E} \times MF \), the set \( MF \) being a set of meaningful formulas of a calculus of unary predicates in the set \( \text{Pred} \) interpreted in a model of STRS.

We let the meanings of negations and implications to be

\[
[[Np]] = U \setminus [[p]]; [[Cpq]] = (U \setminus [[p]]) \cup [[q]],
\]

(10)

where \( [[p]] = \{ u : p(u) \} \) is the meaning of the formula \( p \).

We define the extension \( (I^\mu_\Lambda)^\vee(p) \) of \( I \) at an exact set \( \Lambda \in \mathcal{E} \) and a formula \( p \), relative to the rough inclusion \( \mu \), as follows,

\[
(I^\mu_\Lambda)^\vee(p) = r \Leftrightarrow \mu(\Lambda, [[p]], r). \tag{11}
\]

### 3.4 Regular rough inclusions

We call a rough inclusion \( \mu \) defined on subsets of a universe \( U \), regular, in case \( \mu(X, Y, 1) \) if and only if \( X \subseteq Y \). Thus, regular rough inclusions are extensions of the predicate \( \subseteq \) of part.

We restrict ourselves in this note to regular rough inclusions.

A particular scheme for defining regular rough inclusions, proposed in [13], is based on functions called \( t \)-norms [13], see [14].

It is well known, see, e.g., [14] that in case of an archimedean \( t \)-norm \( T \), i.e., a \( t \)-norm satisfying the inequality \( T(x, x) < x \) for every \( x \in (0, 1) \), a representation,

\[
T(x, y) = g(f(x) + f(y)), \tag{12}
\]

with \( f \) a decreasing continuous function on \([0, 1] \) and \( g \) the pseudo-inverse to \( f \) (see, e.g., [14]) holds.

We define a rough inclusion \( \mu \) on subsets of the universe \( U \) of an instance of STRS by letting,

\[
\mu(X, Y, r) \Leftrightarrow g\left(\frac{|X \setminus Y|}{|X|}\right) = r. \tag{13}
\]

where \( g \) is as in (12). Then clearly, \( \mu \) is regular if and only if \( g^{-1}(1) = 0 \).

To give a specific example, let us turn to the \( \text{\textit{Lukasiewicz t–norm}} T_L(x, y) = \max\{0, x+y-1\} \), in which case we have \( f(u) = 1-u = g(u) \), see, e.g., [14].
Therefore, the corresponding rough inclusion \( \mu_L \) is defined as follows,

\[
\mu_L(X, Y, r) \leftrightarrow 1 - \frac{|X \setminus Y|}{|X|} = r \Leftrightarrow \frac{|X \cap Y|}{|X|} = r, 
\]
and it is regular.

We may observe that the formula (14) reflects the probabilistic way of reasoning and it is explicitly applied in many important notions of data analysis, e.g., in definitions of association rules [1].

Accordingly, the extension \((I^\mu_L)^\vee(p)\) does satisfy the following with respect to negation and implication,

\[
(I^\mu_L)^\vee(Np) = 1 - (I^\mu_L)^\vee(p),
\]
and,

\[
(I^\mu_L)^\vee(Cpq) = \frac{\Lambda \cap \{p\} \cap \{q\}}{|\Lambda|} \leq 1 - (I^\mu_L)^\vee(p) + (I^\mu_L)^\vee(q),
\]
so finally,

\[
(I^\mu_L)^\vee(Cpq) \leq 1 - (I^\mu_L)^\vee(p) + (I^\mu_L)^\vee(q). 
\]

The formula on the right hand side of inequality (17) yields the value of the implication in the Łukasiewicz many–valued logic [4]. We may say that in this case the logic RML is sub–Łukasiewicz many–valued logic.

4 Information systems. Decision (constraint) logic

Information systems are a form of knowledge representation ([10]). Their formal rendering is as pairs of the form \((U, A)\) where \(U\) is the set of objects and \(A\) is the set of attributes understood as mappings \(a : A \rightarrow V_a\), where \(V_a\) is the \(a\)-value set.

Classification \(R\) is obtained in this case as the indiscernibility relation \(IND\) defined by means of,

\[
xINDy \Leftrightarrow a(x) = a(y),
\]
for each \(a \in A\).

Exact sets in this model are unions of equivalence classes of \(IND\). Decision logic (see[10]) starts with descriptors of the form \((a, v)\), where \(v \in V_a\), and builds formulas as the smallest set containing all descriptors and closed under sentential connectives \(\vee, \wedge, \neg\).

Semantics of decision logic is defined by means of the following interpretation of a descriptor,

\[
[[(a, v)]] = \{u \in U : a(u) = v\},
\]
extended recursively over formulas by means of,

\[
[[(\phi \vee \psi)]] = [[\phi]] \cup [[\psi]],
\]
\[
[[(\phi \wedge \psi)]] = [[\phi]] \cap [[\psi]],
\]
\[
[[(\neg \phi)]] = U \setminus [[\phi]].
\]

In what follows, we interpret formulas of decision logics as constraints on objects. This point of view will be useful when we discuss vague statements as statements having a fuzzy–set semantics under rough set formalized constraints.

A variant of an information system in which an attribute, say \(d \notin A\) is selected (and called the decision) is called the decision system [10]. Formally, a decision system is represented as a triple \((U, A, d)\) where \((U, A)\) is an information system and \(d\) is the decision; we will regard in the sequel the pair \((U, A)\) as a model for rough set theory, whereas the decision will serve as the means for producing sets of objects conforming to assumed fuzzy memberships.

5 Towards a Calculus of Vague Statements

Vague statements may be defined as statements of facts of the world that reflect some aspects of the world and objects in it and whose truth can be ascertained up to degree only by a subjective evaluation; a fortiori, each agent reasoning about the given world may have its own evaluation of degree of truth of a given statement depending on its choice of evaluation function called a fuzzy membership function. Formally, such statements with their degrees of truth may be regarded as formulas of the Łukasiewicz [0,1]–valued
logic, considered above. However, one may be aware that there exist a variety of semantics for fuzzy statements.

As an archetypical example, we consider the statement "John is tall". The concept (predicate) "tall" is vague as there is no objective measure of tallness: not only it is not any crisp notion but moreover, its boundary, i.e., the set of objects in a given universe that are classified neither as "tall" nor as "not tall" is not defined crisply – as is the case with the rough set approach – but it is subject to personal evaluation.

Fuzzy set theory approaches this notion with the idea of a pair of the form \((\mu_{\text{tall}}, D_{\text{tall}})\), where \(D_{\text{tall}}\) is a set of values (the domain) of the predicate "tall" and \(\mu_{\text{tall}}\) is a function from the domain \(D_{\text{tall}}\) into the unit interval \([0, 1]\) called the fuzzy membership function. Semantics of the statement "John is tall" is therefore given by the value of \(\mu_{\text{tall}}(\text{height}(John)) \in [0, 1]\), where \(\text{height}\) is the function converting object names into values in the domain \(D_{\text{tall}}\). Conventionally, two modalities are involved here, viz., the necessity that is attached to statements with the value of 1 and possibility that is attached to statements with a positive value.

A program set forth by L.A.Zadeh [19], foresees a system for reasoning under uncertainty that would take as inputs perceptions (statements in natural language about concepts vague as a rule) and it would convert them into constraints on variables leading via constraint propagation to generalized constraints on variables.

We would like to propose here an approach to calculus of vague statements in which constraints derived from natural language statements about objects and their properties are expressed as formulas in the description logic – their meanings being exact sets in a model of rough set theory – while fuzzy statements are evaluated via their cut sets interpreted as sets (rough or exact) in the model universe. The degree of membership of generalized constraint is evaluated via a chosen rough inclusion function by means of a rough mereologic logic.

We give an example of this kind of reasoning. In this example, we apply the Lukasiewicz rough inclusion so our reasoning may be termed in this case frequency based (probabilistic).

5.1 Example

As an exemplary vague perception (of an elementary nature, for simplicity’ sake), we choose the one proposed in [19] (where it is a part of a complex perception),

(P) "Carol has two children: Robert, who is in mid-twenties, and Helen, who is in mid-thirties",

that is subject to the vague query,

(Q) "How old is Carol"?

To produce a generalized constraint on the variable "age_of_Carol", we begin with a model of the rough set theory in the form of a decision system \(Age\) presented in Table 1, where \(c\_n\) stands for "number of children", \(c\_i\) stands for "child number \(i\) age", and \(d\) denotes the decision attribute "Age".

The interpretation of the concept "Old" is given as,

\[
\mu_{\text{Old}}(x) = \begin{cases} 
0.02(x - 30) & \text{for } x \in [30, 60] \\
0.04(x - 60) + 0.6 & \text{else}
\end{cases}
\]  

(23)

The interpretation of the concept \(\mu_{20}\), "in mid twenties", is,

\[
\mu_{20}(x) = \begin{cases} 
0.25(x - 20) & \text{for } x \in [20, 24] \\
1 & \text{for } x \in [24, 26]
\end{cases}
\]

\[
1 - 0.25(x - 26) & \text{for } x \in [26, 30]
\]  

(24)

The interpretation of the concept \(\mu_{30}\), "in mid thirties", is,

\[
\mu_{30}(x) = \begin{cases} 
0.25(x - 30) & \text{for } x \in [30, 34] \\
1 & \text{for } x \in [34, 36]
\end{cases}
\]

\[
1 - 0.25(x - 36) & \text{for } x \in [36, 40]
\]  

(25)
For a vague concept $X$, interpreted as a pair $(\mu_X, D_X)$, and $c \in [0, 1]$, we denote with the symbol $X_c$ the set $\{x \in D_X : \mu_X(x) \geq c\}$ (the $c$-cut set of $X$).

We construct the generalized constraint on $\textit{age}_\text{-of}_\text{-Carol}$ as a function $f : [0,1]^3 \rightarrow [0,1]$ that would specify for a given vector $(\alpha, \beta, \gamma) \in [0,1]^3$, the truth degree of the resulting predicate induced by the $\alpha$ cut set measured over the exact set resulting from constraints on objects with age $\beta, \gamma$.

Given $(\alpha, \beta, \gamma) \in [0,1]^3$, we form in this case three sets: $\textit{Old}_\alpha, \textit{In\text{-}mid\text{-}twenties}_\beta, \textit{In\text{-}mid\text{-}thirties}_\gamma$ being cut sets of respective vague notions. The last two sets induce constraints in the decision logic language of the decision system $\textit{Age}$. The notation $(a, \in [u, v])$ is the shortcut for the disjunction $\bigvee_{w \in [u,v]} (a, w)$.

We give an example of constraints in case $\beta = 0.5 = \gamma$. In this case, we have,

$$\textit{In\text{-}mid\text{-}twenties}_\beta = [23, 27], \quad \textit{In\text{-}mid\text{-}thirties}_\gamma = [33, 37]. \quad (27)$$

### 5.1.2 Constraints on objects

The constraint is the disjunction of the following formulas,

$$\begin{align*}
(c_{-1}, 2) \wedge (c_{-1}, 1) & \in [23, 27] \wedge (c_{-2}, 1) \in [33, 37] \\
(c_{-1}, 3) \wedge (c_{-1}, 1) & \in [23, 27] \wedge (c_{-2}, 1) \in [33, 37] \\
(c_{-1}, 3) \wedge (c_{-1}, 1) & \in [23, 27] \wedge (c_{-2}, 3) \in [33, 37] \\
(c_{-1}, 3) \wedge (c_{-2}, 2) & \in [23, 27] \wedge (c_{-2}, 3) \in [33, 37]
\end{align*} \quad (28)$$

The meaning of the constraint (28) is the exact set,

$$\Lambda = \{4, 8, 12, 18, 20\}. \quad (29)$$

### 5.1.3 Constraints on decision

We set $\alpha = 0.6$, and thus the cut set for $\textit{Old}$ is,

$$\textit{Old}_{0.6} = \{5, 6, 7, 8, 9, 12, 13, 14, 15, 17, 18, 20\}. \quad (30)$$

Following our convention, we may express this set as the meaning of the predicate $\textit{old}_{0.6}(x)$ that is true if and only if $x \in \textit{Old}_{0.6}$.

### 5.1.4 Evaluation of truth degree

For the chosen values of $\alpha$, $\beta$, $\gamma$, we evaluate the truth degree of the predicate $\textit{old}_{0.6}(x)$ with respect to the exact set $\Lambda$, by applying the intensional logic $\text{RML}^{\nu_L}$,

$$(I_\Lambda^{\nu_L})(\textit{old}_{0.6}(x)) = \frac{|\Lambda \cap [\textit{old}_{0.6}(x)]|}{|\Lambda|} = 0.8. \quad (31)$$

We may say, that the precise formula $f(0.6, 0.5, 0.5) = 0.8$ can be rendered informally as the statement that: "Carol is at least 60 years old" is true to degree 0.8 under the assumed interpretation of vague statements "Carol has a child in mid-twenties" and "Carol has a child in mid-thirties".

The fuzzy set representing the above vague statement, whose fuzzy membership function is given by $f$, is the set of triples of the form $J_1 \times J_2 \times J_3$, where $J_1$ is the interval of the form $[j_1, 70] \subseteq D_{\textit{Old}}$, $J_2$ is the interval of the form $[j_2(1), j_2(2)] \subseteq D_{20}$, and $J_3$ is the interval of the form $[j_3(1), j_3(2)] \subseteq D_{30}$.
Final Remarks

The presented approach to calculus of perceptions, can be regarded as a new formulation of rough–fuzzy hybrid approach to uncertainty. Its applications, involving rough mereological logics, will be studied both from theoretical as well as application points of view.

References


