Exploring Gaussian and Triangular Primary Membership Functions in Non-Stationary Fuzzy Sets

S. Musikasuwan and J.M. Garibaldi
Automated Scheduling, Optimisation and Planning Group
University of Nottingham, Nottingham, NG8 1BB, UK
{svm,jmg}@cs.nott.ac.uk

Abstract

Non-stationary fuzzy sets have previously been introduced to allow the modelling of variation in the membership value associated with a given value of the base variable of a fuzzy set. In order to explore how the form of the primary membership function (MF) affects the inference process within a non-stationary fuzzy system, a study was carried out on a fuzzy system implementing the XOR problem, in which either Gaussian or Triangular MFs were employed. Investigations were carried out into different 'perturbation functions' and different type of variation. These non-stationary fuzzy systems were also compared to conventional type-2 fuzzy systems featuring equivalent 'Footprints of Uncertainty'. It was observed that non-stationary fuzzy systems using a Sine-based perturbation function produced almost the same results as interval type-2 systems (except for centre variation of Gaussian MFs). In other cases, more complex relationships between the uncertainties obtained with the two methods were observed. This observation requires further investigation.

Keywords: Interval Type-2 Fuzzy Sets, Non-Stationary Fuzzy Sets, Gaussian Membership Function, Triangular Membership Function.

1 Introduction

In 1975, Zadeh [1] proposed 'fuzzy sets with fuzzy membership functions' as an extension of the concept of an ordinary, i.e. type-1, fuzzy set and went on to define fuzzy sets of type n, n = 2, 3, ..., for which the membership function ranges over fuzzy sets of type n - 1 to model the uncertainties and minimize their effects.

Type-2 fuzzy sets are characterized by three-dimensional MFs. The membership grade for each element of a type-2 fuzzy set is a fuzzy set in [0,1]. The additional third dimension provides additional degrees of freedom to capture more information about the represented term. Type-2 fuzzy sets are useful in circumstances where it is difficult to determine the exact membership function for a fuzzy set, which is useful for incorporating uncertainties. However, the use of type-2 fuzzy sets in practice has been limited due to the significant increase in computational complexity involved in their implementation. Recently, Mendel has introduced a concept known as the *footprint of uncertainty* which provides a useful verbal and graphical description of the uncertainty captured by any given type-2 set. Mendel has particularly concentrated on a restricted class of general type-2 fuzzy sets known as *interval type-2 fuzzy sets* [2]. Interval type-2 sets are characterised by having secondary membership functions which only take the values 0 or 1. This restriction greatly simplifies the computational requirements involved in performing inference with type-2 sets. Mendel and John developed a simple method to derive union, intersection, and complement, and computational algorithms for type reduction (necessary for type-2 defuzzification) [3].

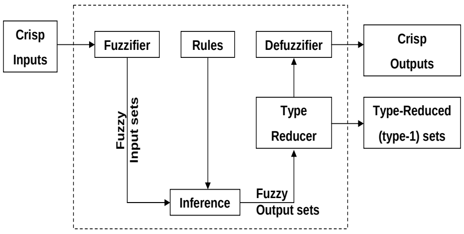


Figure 1: Mechanisms of a type-2 Fuzzy System

Fuzzy systems which are used for representing and inferring with knowledge that is imprecise, uncertain, or unreliable consist of four main interconnected components: *rules, fuzzifier, inference engine, and output processor*. Once the rules have been established, fuzzy systems can be viewed as a non-linear mapping from inputs to outputs. Type-1 Fuzzy system use only type-1 fuzzy sets and a Fuzzy system which uses at least one type-2 fuzzy set is called a type-2 fuzzy system. Fig. 1 shows the mechanisms of a type-2 fuzzy system. The interested reader is particularly referred to [2] for a summary tutorial and for more details.

All humans, including *experts*, exhibit variation in their decision making. Variation may occur among the decisions of a panel of human experts (inter-expert variability), as well as in the decisions of an individual expert over time (intra-expert variability). Up to now it has been an implicit assumption that expert systems, including fuzzy expert systems (FESs), should not exhibit such variation. While type-2 fuzzy sets capture the concept of introducing uncertainty into membership functions by introducing a range of membership values associated with each value of the base variable, they do not capture the notion of variability — a type-2 fuzzy inference system will always produce the same output(s) (albeit a type-2 set with an implicit representation of uncertainty) for given the same input(s). Garibaldi et al [4]-[8] have been investigating the incorporation of variability into decision making in the context of FESs in medical domain. In this work, Garibaldi proposed the notion of 'non-deterministic fuzzy reasoning' in which variability is introduced into the membership functions of a fuzzy system through the use of random alterations to the parameters of the generating function(s). Later, Garibaldi and Musikasuwan [9, 10] extended and formalised this notion through the introduction of a notion that they termed a 'non-stationary fuzzy set'. In the research presented here, we explored non-stationary fuzzy systems using two difference shapes of primary MFs, i.e., Gaussian and Triangular MFs. The experiments were designed by constructing the interval type-2 and non-stationary fuzzy systems using Gaussian or Triangular MFs as the primary MFs in a system to predict the results of the standard XOR problem. Various inputs were presented to the fuzzy systems and were propagated through the inference rules to form the output (consequent) sets. The lower, mean, upper, and interval of the output for each case were computed and recorded.

2 Non-Stationary Fuzzy Sets and Systems

As mentioned in Section 1, Garibaldi proposed the notion of 'non-deterministic fuzzy reasoning' in which variability is introduced into the membership functions of a fuzzy system through the use of random alterations to the parameters of the generating function(s). Later Garibaldi and Musikasuwan extended and formalised this notion through the introduction of a notion that they termed a *non-stationary fuzzy set* [9, 10]. Fig. 2 shows how the inferencing mechanisms might be implemented in such non-stationary FESs. At each instantiation a non-stationary fuzzy system operates as type-1 fuzzy system. However, each particular instantiation may vary from the previous one by a small amount (caused by applying the perturbation function to the parameter of the MFs), to produce slightly different output(s). Hence the output(s) are recorded and then the process will be repeated some number of times. Once all repeated processes have been completed, the final outputs will be calculated. Again, there is a range of options for how to determine the final output from the repeated runs.

Definition 1 *A non-stationary fuzzy set, denoted* $\dot{A}$, *is characterised by a membership function,* $\mu_{\dot{A}}(x,t)$, *where* $(x) \in X$ *and* $\mu_{\dot{A}}(x,t) \in [0,1]$ *and* t *is a free variable,* time — *the time at which the fuzzy set is instantiated, i.e. as in Equation 1,*

$$\dot{A} \;=\; \int_{x \in X} \mu_{\dot{A}}(x,t)/x \;,\; \mu_{\dot{A}} \in [0,1] \qquad (1)$$

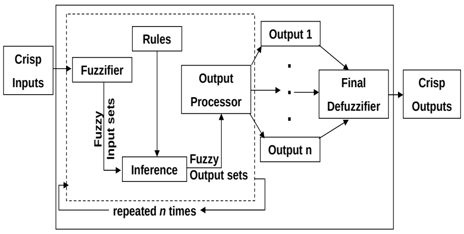


Figure 2: Proposed mechanisms of a Non-Stationary Fuzzy System

Any membership function may be used. In practice, of course, only a few alternative membership functions are found in type-1 fuzzy sets, namely piecewise linear including left-slope, triangular, right-slope, and trapeziodal; gaussian; and sigmoidal. Three main alternative kinds of non-stationarity have been proposed [10]:

- Variation in location — i.e. small alterations to the centre point of the primary membership function
- Variation in slope — i.e. small alterations to the width of the primary membership function
- Noise variation — i.e. making small alterations (vertically) in the value of the membership function.

At any given moment of time, i.e. in any specific instantiation, a non-stationary fuzzy set will instantiate a standard type-1 fuzzy set.

2.1 Perturbation Functions

A function, termed the *perturbation function*, is a function of time that will generate small changes in the base membership function. In theory, this could be a true random function — i.e. the membership function parameter could be a true random variable: hence the terminology of *non-stationary* fuzzy sets. In general, it would be appear that any function of time might be used as the perturbation function, where the only restriction is that membership function remains in bounds. Given that any measurement of time is arbitrary and relative, the actual set of functions that might be useful in practice is more restrictive. Any units might be used for time, t, but the most natural would be to express time in seconds (s), in the absence of any good reason not to. Again, given that any physical notion of time is relative, any arbitrary point in time might be chosen as zero.

A few possibilities for perturbation functions in practice are:

- sine / cosine based, e.g.:

$$f(t) = sin(\omega t) \tag{2}$$

- pseudo-random, e.g.:

$$s(t+1) = (25{,}214{,}903{,}917s(t)+11)\ mod\ 2^{48}$$
$$f(t) = \frac{s(t+1)-2^{47}}{2^{47}} \tag{3}$$

- differential time-series, e.g.:

$$f(x,t) \rightarrow \frac{dx(t)}{dt} = \frac{0.2x(t-\tau)}{1+x^{10}(t-\tau)} - 0.1x(t) \tag{4}$$

3 Experiments

In order to investigate the effect of different primary membership shapes in non-stationary fuzzy sets, Gaussian and Triangular MFs were compared with interval type-2 sets. As stated earlier, this paper is continued from [9] and this section focuses on constructing fuzzy systems to solve the standard XOR problem.

In this study, fuzzy systems were constructed to predict the output of truth value where both input variables can take any value in the range of [0,1]. All fuzzy systems consist of two input variables which are *Input1* and *Input2*, one output variable which is *Output*, and four rules. Each variable consist of 2 Gaussian or Triangular MFs which are *Low* and *High*. The following 4 rules are used for all fuzzy systems. These rules are constructed based on the standard XOR problem.

1. IF *Input1* is *Low* AND *Input2* is *Low* THEN *Output* is *Low*
2. IF *Input1* is *Low* AND *Input2* is *High* THEN *Output* is *High*
3. IF *Input1* is *High* AND *Input2* is *Low* THEN *Output* is *High*
4. IF *Input1* is *High* AND *Input2* is *High* THEN *Output* is *Low*

There are three kinds of perturbation function that were used in this study, as follows:

- Sine based function (where $\omega = 127$) (Eq. 2)
- Uniformly distributed function (Eq. 3)
- Normally distributed random function

Sine based and Uniformly distributed functions return numbers in the range $[-1,1]$, while the third (the Matlab *randn* function) returns real numbers sampled from a Normal distribution with mean zero and standard deviation one.

3.1 Gaussian Primary Membership Functions

The primary Gaussian MFs as shown in Fig. 3 were used and two kinds of variation were investigated, i.e. *centre variation* and *width variation*.

3.1.1 Non-stationary Fuzzy Systems

In both case of centre and width variations, 3 different fuzzy systems (described by perturbation function used to generate MFs, i.e.; Sine function, Uniformly distributed, and Normally distributed) were designed with two inputs (antecedents), one output (consequent), two Gaussian MFs for each antecedents and consequent, and four rules. All terms (two inputs and one output) had two Gaussian membership functions, corresponding to meanings of *Low* and *High*. *Low* membership functions all had centre 0.1, *High* membership functions all had centre 0.9. Finally, the initial widths for all MFs for all terms were 0.5. Note that the parameters of the primary membership functions were chosen completely arbitrarily, since we do not consider their precise values to be of any importance. The purpose of the study is purely to explore the similarities and differences between the non-stationary fuzzy systems and the equivalent interval type-2 systems in each case.

The four input vectors, (0.25,0.25) (0.25,0.75) (0.75,0.25) and (0.75,0.75), were presented to the system and each time the non-stationary fuzzy sets were generated by replacing centre (c) or width (σ) with $c = c + 0.05f(t)$ or $\sigma = \sigma + 0.05f(t)$ (where $f(t)$ represents the chosen *perturbation function*), respectively. To clarify, the non-stationary fuzzy sets were regenerated for *each* input vector. This process was repeated a fixed number of times (30 times for this study). As an aside, note that it would appear to be a perfectly acceptable design choice to generate the fuzzy sets of the non-stationary system once before presenting the four input vectors, and then to regenerate once again for the next set of four input vectors. We are continuing to investigate such alternative design choices in ongoing work.

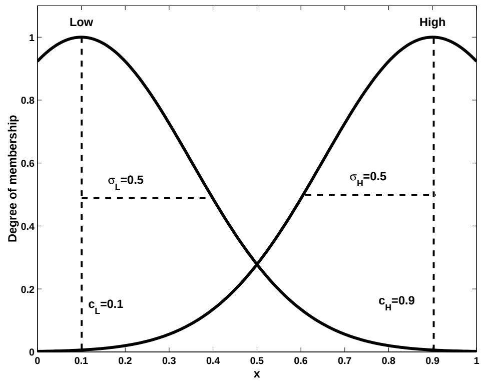


Figure 3: Gaussian primary membership functions used in the experiments

3.1.2 Interval type-2 Fuzzy Systems

Two interval type-2 systems were also designed with 2 inputs (antecedents), 1 output (consequent), 2 Gaussian MFs for each antecedent and consequent, and four rules. The membership functions all had the same centre and width parameters as described above.

In the type-2 system, the footprint of uncertainty of the type-2 MFs were created by deviating the parameters of the original type-1 MFs by a percentage of the universe of discourse of the variables that they were associated to. Two different methods were used to create these type-2 MFs: by varying the centre point, and varying the width around the original type-1 MF. In the case of varying the centre, the centre of lower and upper bounds MFs were defined by shifting the initial centre point both left and right for 5% of universe of discourse of variable that MF belongs to, respectively, as follows:

- Centre of lower MF = $c \pm 0.05$

Similarly, in the case of varying the width, the width of lower and upper bounds MFs were defined by shifting the initial width both left and right for 5% of universe of discourse of variable that MF belongs to, respectively, as follows:

- Width of lower MF = $\sigma \pm 0.05$

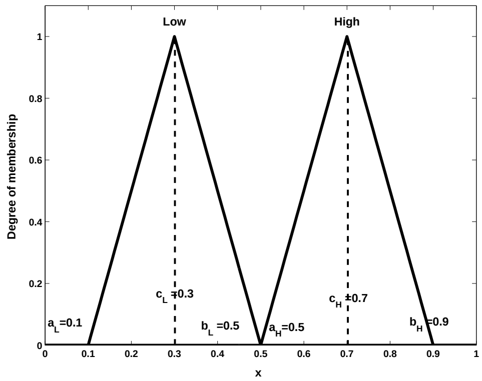


Figure 4: Primary triangular membership function used in the experiments

3.2 Triangular Primary Membership Functions

For the case of triangular membership functions, four kinds of variation were investigated, i.e. *centre variation*, *begin-point variation*, *end-point variation*, and *begin & end point variation*.

3.2.1 Non-stationary Fuzzy Systems

The triangular shapes used throughout this case study to represent membership functions are shown in Fig. 4. The non-stationary fuzzy sets were then generated by replacing the begin-point a and/or end-point b, or centre-point c in Fig. 4 with $a = a + 0.05f(t)$, $b = b + 0.05f(t)$, and $c = c + 0.05f(t)$, where $f(t)$ represents the chosen *perturbation function*. This process was again repeated 30 times. In all cases of variation, 3 different fuzzy systems (described by perturbation function used to generate MFs, i.e.; Sine function, Uniformly distributed, and Normally distributed) were designed with two inputs (antecedents), one output (consequent), two triangular MFs for each antecedents and consequent, and four rules. All terms (two inputs and one output) had two triangular membership functions, corresponding to meanings of *Low* and *High*. *Low* membership functions all have ordinary centre $c = 0.3$, $a = 0.1$, and $b = 0.5$; *High* membership functions all had ordinary centre $c = 0.7$, $a = 0.5$, and $b = 0.9$.

3.2.2 Interval type-2 Fuzzy Systems

Similarly, eight interval type-2 systems were also designed with 2 inputs (antecedents), 1 output (consequent), 2 triangular MFs for each antecedent and consequent, and four rules. The membership functions all had the same parameters as described above.

In the type-2 system, the footprints of uncertainty of the type-2 MFs were created by deviating the parameters of the original type-1 MFs by a percentage of the universe of discourse of the variables that they were associated with. Four methods were used to create these type-2 MFs, to match those of the non-stationary systems.

(1) Varying the centre point of the original type-1 MF. The centre of lower and upper bounds MFs were defined by shifting the initial centre c both left and right for 5% of the universe of discourse of the variable's MF, as follows:

- Centre of lower and upper MF = $c \pm 0.05$

(2) Varying the begin-point of the original type-1 MF. The begin-point of lower and upper bounds MFs were defined by shifting the initial begin-point a both left and right for 5% of the universe of discourse of the variable's MF, as follows:

- begin-point of lower and upper MF = $a \pm 0.05$

(3) Varying the end-point of the original type-1 MF. The end-point of lower and upper bounds were defined by shifting the initial end-point b both left and right for 5% of the universe of discourse of the variable's MF, as follows:

- end-point of lower and upper MF = $b \pm 0.05$

(4) Varying both begin and end points around the original type-1 MF. The begin and end points of lower and upper bounds MFs were defined by shifting the initial begin and end points a and b both left and right for 2.5% of the variable's MF, as follows:

- begin-point of lower and upper MF = $a \pm 0.025$
- end-point of lower and upper MF = $b \pm 0.025$

4 Methods

After all systems had been constructed, they were used to predict the output of each of the four input vectors ((0.25,0.25) (0.25,0.75) (0.75,0.25) and (0.75,0.75)). The lower, mean, upper, and interval of the results were computed and recorded.

Table 1: Lower, Mean and Upper Bounds for Gaussian Membership Functions

Variation	*Type*	*Perturbation*	*Input 1 (0.25,0.25)*			*Input 2 (0.25,0.75)*			*Input 3 (0.75,0.25)*			*Input 4 (0.75,0.75)*		
			Lower	*Mean*	*Upper*	*Lower*	*Mean*	*Upper*	*Lower*	*Mean*	*Upper*	*Lower*	*Mean*	*Upper*
Centre	*Type-2*	*Interval*	0.3687	0.3956	0.4224	0.5780	0.6050	0.6320	0.5780	0.6050	0.6320	0.3687	0.3956	0.4224
	Non-Stationary	*Normal*	0.3853	0.3937	0.4020	0.5970	0.6056	0.6141	0.5970	0.6056	0.6141	0.3853	0.3937	0.4020
		Uniform	0.3932	0.3933	0.4033	0.5791	0.6061	0.6331	0.5791	0.6061	0.6331	0.3932	0.3933	0.4033
		Sine	0.3854	0.3939	0.4033	0.5795	0.6065	0.6335	0.5795	0.6065	0.6335	0.3854	0.3939	0.4033
Width	*Type-2*	*Interval*	0.3836	0.3921	0.4007	0.5993	0.6079	0.6164	0.5993	0.6079	0.6164	0.3836	0.3921	0.4007
	Non-Stationary	*Normal*	0.3735	0.3911	0.4088	0.5912	0.6089	0.6265	0.5912	0.6089	0.6265	0.3735	0.3911	0.4088
		Uniform	0.3734	0.3933	0.4097	0.5903	0.6067	0.6267	0.5903	0.6067	0.6267	0.3734	0.3933	0.4097
		Sine	0.3732	0.3923	0.4098	0.5902	0.6078	0.6268	0.5902	0.6078	0.6268	0.3732	0.3923	0.4098

In the case of interval type-2 systems, the lower and upper outputs were obtained directly [3], and the mean is simply the average of lower and upper bounds. In the case of non-stationary systems, for Sine and Uniform perturbation functions, the lower bound values were derived from minimum output value, the upper bound values were derived from maximum output value, and the mean were derived from average of the output value from 30 the repeated runs. Finally, the interval of the outputs were derived by computing the length between the lower and upper output values.

For the systems generated by Normally distributed random number (only), the lower and upper bounds are derived from $m \pm s$, where m is the mean of the outputs over time and s is the standard deviation. Finally, the outputs of four input sets ([(0.25,0.25) (0.25,0.75) (0.75,0.25) (0.75,0.75)]) were presented in Section 5.

5 Results

In the case of Gaussian MFs, with centre variation, the lower and upper bounds of the obtained values and the final centroid output values for all 4 fuzzy systems are shown in Table 1. The same information is also presented for width variation.

Similarly, in case of Triangular MFs, the lower and upper bounds predicted values and the final centroid output values for all systems are also shown in Table 2 — for centre variation; begin point variation; for end point variation; and for both begin and end points variation, respectively.

The length of each results interval was calculated and recorded. In case of Gaussian primary MF, Fig. 5 shows the plots of mean of intervals for the non-stationary systems together with interval type-2 fuzzy systems. Similarly, in case of Triangular primary MF, the plots of mean of intervals for the non-stationary systems together with interval type-2 fuzzy systems are shown in Fig. 6.

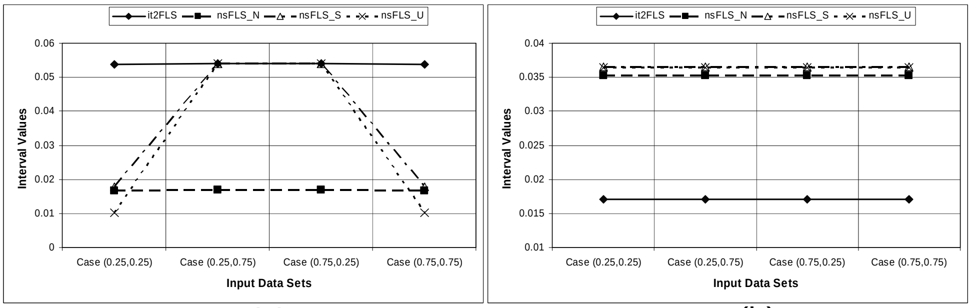
6 Discussion

The class of a type-2 fuzzy set is determined by the secondary membership function. In comparison, the class of a non-stationary fuzzy set is determined both by which kind of non-stationarity used (variation in location, variation in slope or noise variation) and by the form of perturbation function used to deviate the primary membership function — in this study we have used Normally distributed, Uniformly distributed, and Sine based perturbation functions applied to both variation in location and variation in slope. It should be noted, therefore, that herein lies a subtle difference between non-stationary fuzzy sets used in this paper and type-2 fuzzy sets. In the non-stationary fuzzy sets used here, the perturbation function acts *horizontally* across the universe of discourse; in type-2 fuzzy sets the secondary membership functions are defined *vertically* along the membership value μ. For non-stationary fuzzy sets featuring 'noise variation', the perturbation function acts *vertically*. Of course, different perturbation functions can still be used and, thus, such non-stationary fuzzy sets might provide a more 'direct' comparison with type-2 fuzzy sets. Again, we are further exploring these areas.

Turning to the results obtained for the interval of outputs obtained in the experiments carried out. In Fig. 5 (Gaussian primary MFs), it can be seen in (b) (width variation) that the output interval is constant for the type-2 system and for all the non-stationary systems. However, all the non-stationary systems exhibit (the same) larger output interval. This is a curious finding. In

Table 2: Lower, Mean and Upper Bounds for Triangular Membership Functions

Vari-ation	*Type*	*Perturb-ation*	*Input 1 (0.25,0.25)*			*Input 2 (0.25,0.75)*			*Input 3 (0.75,0.25)*			*Input 4 (0.75,0.75)*		
			Lower	*Mean*	*Upper*	*Lower*	*Mean*	*Upper*	*Lower*	*Mean*	*Upper*	*Lower*	*Mean*	*Upper*
Centre point	*Type-2*	*Interval*	0.2833	0.2981	0.3129	0.6871	0.7000	0.7129	0.6871	0.7000	0.7129	0.2871	0.3019	0.3167
	Non-Stationary	*Normal*	0.2843	0.2986	0.3130	0.6881	0.7002	0.7123	0.6881	0.7002	0.7123	0.2874	0.3017	0.3161
		Uniform	0.2835	0.2997	0.3124	0.6873	0.7004	0.7124	0.6873	0.7004	0.7124	0.2873	0.3009	0.3160
		Sine	0.2846	0.2989	0.3129	0.6871	0.7000	0.7129	0.6871	0.7000	0.7129	0.2871	0.3010	0.3166
Begin point	*Type-2*	*Interval*	0.2828	0.3004	0.3180	0.6825	0.7003	0.7180	0.6825	0.7003	0.7180	0.2825	0.3000	0.3175
	Non-Stationary	*Normal*	0.2812	0.3007	0.3203	0.6806	0.7003	0.7201	0.6806	0.7003	0.7201	0.2819	0.3001	0.3183
		Uniform	0.2829	0.3006	0.3173	0.6826	0.7006	0.7173	0.6826	0.7006	0.7173	0.2826	0.3005	0.3168
		Sine	0.2828	0.3001	0.3180	0.6825	0.7000	0.7180	0.6825	0.7000	0.7180	0.2825	0.2999	0.3175
End point	*Type-2*	*Interval*	0.2825	0.3000	0.3175	0.6820	0.6998	0.7175	0.6820	0.6998	0.7175	0.2820	0.2996	0.3172
	Non-Stationary	*Normal*	0.2819	0.3001	0.3184	0.6775	0.6992	0.7210	0.6775	0.6992	0.7210	0.2788	0.3004	0.3221
		Uniform	0.2826	0.3006	0.3169	0.6822	0.7005	0.7168	0.6822	0.7005	0.7168	0.2822	0.3004	0.3166
		Sine	0.2825	0.2999	0.3175	0.6820	0.6998	0.7175	0.6820	0.6998	0.7175	0.2820	0.2998	0.3172
Begin & End point	*Type-2*	*Interval*	0.2827	0.3002	0.3177	0.6823	0.7000	0.7177	0.6823	0.7000	0.7177	0.2823	0.2998	0.3173
	Non-Stationary	*Normal*	0.2819	0.3003	0.3187	0.6814	0.7001	0.7188	0.6814	0.7001	0.7188	0.2815	0.2999	0.3183
		Uniform	0.2828	0.3006	0.3170	0.6825	0.7005	0.7170	0.6825	0.7005	0.7170	0.2824	0.3005	0.3167
		Sine	0.2827	0.3000	0.3177	0.6823	0.7000	0.7177	0.6823	0.7000	0.7177	0.2823	0.2999	0.3173



(a) (b)

Figure 5: Means of the intervals of the outputs for Gaussian non-stationary and interval type-2 fuzzy systems (a) centre variation (b) width variation

contrast, it (a) (centre variation), the picture is very much more complex. The type-2 system has a constant output interval, as does the Normally distributed non-stationary system; however, the Normally distributed non-stationary system now has a *smaller* output interval. Furthermore, the output interval of the Uniform and Sine non-stationary system varies between that corresponding to the Normally distributed non-stationary system for 'symmetric' inputs (0.25,0.25) and (0.75,0.75), and corresponding to the type-2 system for the non-symmetric inputs (0.25,0.75) and (0.75,0.25). Again, these findings are curious.

In Fig. 6 (Triangular primary MFs), again the relationships are far from straight-forward. For begin and end-point variation ((b) and (c)), the output intervals appear to be non-symmetrical with the inputs. This is perhaps not surprising, as the membership functions are being altered in a non-symmetrical manner. However, the absolute value of output interval for the Normally distributed non-stationary systems is larger and the non-symmetry is more exaggerated. For the case of centre variation (Fig. 6 (a)), all systems have approximately the same value of interval, which varies according to the input values. For begin and end points (i.e. width) variation (Fig. 6 (d)), the interval of Normally distributed non-stationary systems are larger than all others. We are unable to draw any definitive conclusions from the results obtained here. For all cases except centre variation of Gaussian primary MFs, the Sine perturbation function produces results which are *very* close to the interval type-2 systems. Why if should be different for the one case, we are currently at a loss to explain.

Non-stationary fuzzy sets provide a relatively straight-forward mechanism for carrying out inference with fuzzy sets that are *uncertain* in some way. Clearly non-stationary systems are not di-

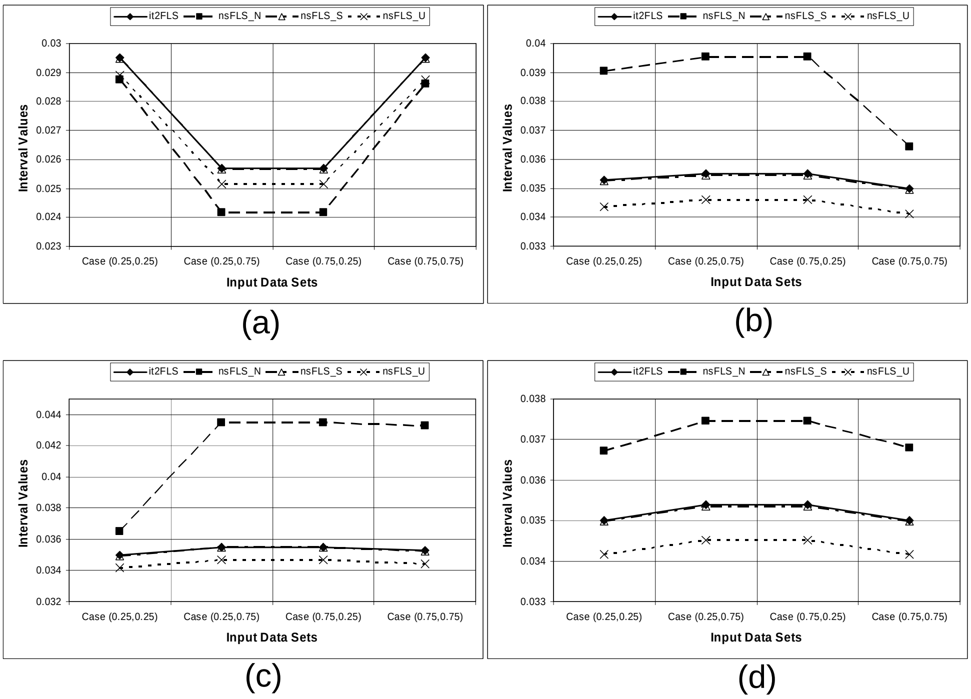


Figure 6: Means of the intervals of the outputs for Triangular non-stationary and interval type-2 systems (a) centre variation (b) begin-point variation (c) end-point variation and (d) begin & end points variation

rect equivalents of type-2 systems. However, non-stationary fuzzy systems may provide a mechanism whereby a form of fuzzy reasoning which *approximates* (in some meaning of the word) general type-2 fuzzy inference in a simple, fast and computationally efficient manner. We are continuing investigations into the relationship between the two frameworks (non-stationary systems and type-2 systems) in order to explore this approximation of interval and general type-2 inference further.

Acknowledgements

This work was supported by the Royal Thai Government.

References

[1] L. Zadeh. The concept of a linguistic variable and its application to approximate reasoning - I,II,III. *Information Sciences*, vol. 8;8;9, pp. 199-249;301-357;43-80, 1975.

[2] J. Mendel. Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions. NJ: Prentice-Hall, 2001.

[3] J. Mendel and R. John. Type-2 fuzzy sets made simple. *IEEE Transactions on Fuzzy Systems*, vol. 10, pp. 117-127, 2002.

[4] T. Ozen and J.M. Garibaldi. Investigating adaptation in type-2 fuzzy logic systems applied to umbilical acid-base assessment. In *Proc. of European Symposium on Intelligent Technologies, Hybrid Systems and Their Implementation on Smart Adaptative Systems*, Oulu, Finland, June 2003.

[5] T. Ozen, J.M. Garibaldi, and S. Musikasuwan. Preliminary investigations into modelling the variation in human decision making. In *Proc. of Information Processing and Management of Uncertainty in Knowledge Based Systems*, Perugia, Italy, July 2004.

[6] T. Ozen, J.M. Garibaldi, and S. Musikasuwan. Modelling the variation in human decision making. In *Proc. of Fuzzy Sets in the Heart of Canadian Rockies-NAFIPS 2004*, Banff, Canada, June 2004.

[7] T. Ozen and J.M. Garibaldi. Effect of type-2 fuzzy membership function shape on medelling variation in human decision making. In *Proc. of IEEE International Conference on Fuzzy Systems*, Budapest, Hungary, July 2004.

[8] T. Ozen and J.M. Garibaldi. Nondeterministic fuzzy reasoning. Submitted to *IEEE Transactions on Fuzzy Systems*, 2005.

[9] J.M. Garibaldi and S. Musikasuwan. The Association between Non-Satationary and Interval Type-2 Fuzzy Sets: A Case Study. In *Proc. of IEEE International Conference on Fuzzy Systems*, Reno, USA, May 2005.

[10] S. Musikasuwan and J.M. Garibaldi. Non-Stationary Fuzzy Sets. In prep. for *IEEE Transactions on Fuzzy Systems*, 2005.