# Neuro-Dynamic Programming An Overview

Dimitri Bertsekas
Dept. of Electrical Engineering
and Computer Science
M.I.T.

May 2006

# BELLMAN AND THE DUAL CURSES

- Dynamic Programming (DP) is very broadly applicable, but it suffers from:
  - Curse of dimensionality
  - Curse of modeling
- We address “complexity” by using approximations (based loosely on parametric/neural architectures)
- Unlimited applications in planning, resource allocation, stochastic control, discrete optimization
- Application is an art … but guided by substantial theory

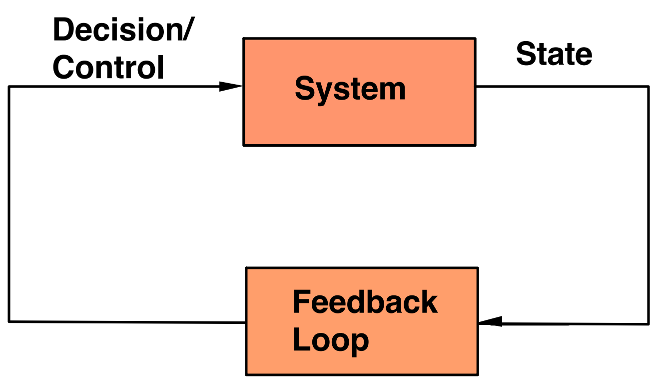
# OUTLINE

- Main NDP framework
- Discussion of two classes of methods:
  - Actor-critic methods/LSPE
  - Rollout algorithms
- Connection between rollout and Model Predictive Control (MPC)
- Book references:
  - Neuro-Dynamic Programming (Bertsekas + Tsitsiklis)
  - Reinforcement Learning (Sutton + Barto)
  - Dynamic Programming: 3rd Edition (Bertsekas)
- Papers can be downloaded from

  http://web.mit.edu/dimitrib/www/home.html

# DYNAMIC PROGRAMMING / DECISION AND CONTROL

- **Main ingredients:**
  - Dynamic system; state evolving in discrete time
  - Decision/control applied at each time
  - Cost is incurred at each time
  - There may be noise & model uncertainty
  - There is state feedback used to determine the control



# ESSENTIAL TRADEOFF CAPTURED BY DP

- **Decisions are made in stages**
- **The decision at each stage:**
  - **Determines the present stage cost**
  - **Affects the context within which future decisions are made**
- **At each stage we must trade:**
  - **Low present stage cost**
  - **Undesirability of high future costs**

# KEY DP RESULT: BELLMAN'S EQUATION

- Optimal decision at the current state minimizes the expected value of

  Current stage cost + Future stages cost starting from the next state (using opt. policy)

- Extensive mathematical methodology
- Applies to both discrete and continuous systems (and hybrids)
- Dual curses of dimensionality/modeling

# KEY NDP IDEA

- Use one-step lookahead with an “approximate” cost
- At the current state select decision that minimizes the expected value of

  Current stage cost + Approximate future stages cost starting from the next state

- Important issues:
  - How to construct the approximate cost of a state
  - How to understand and control the effects of approximation

# METHODS TO COMPUTE AN APPROXIMATE COST

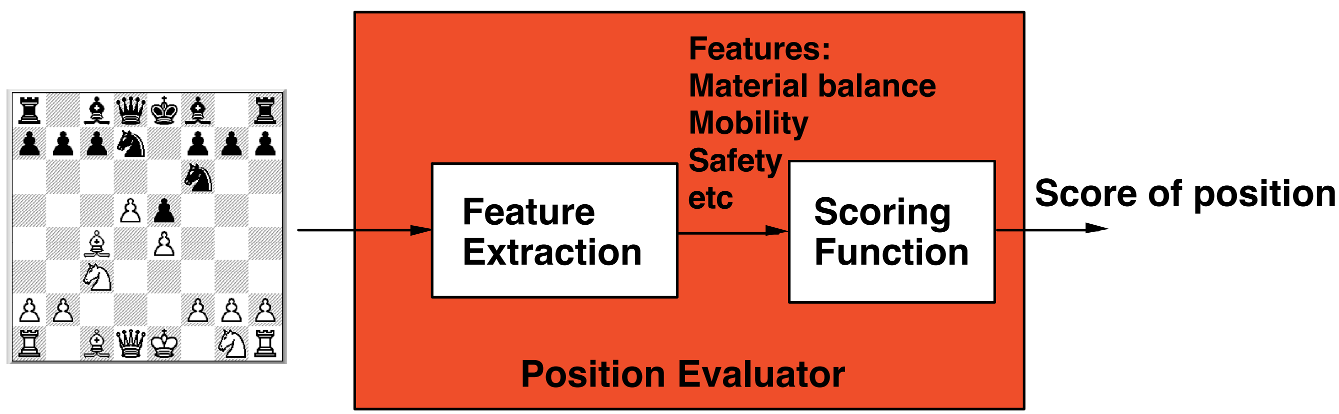
- Parametric approximation algorithms (off-line)
  - Use a functional approximation to the optimal cost function
  - Select the weights of the approximation - connection with "neural networks"
  - One possibility: Hand-tuning, and trial and error
  - Systematic DP-related policy and value iteration methods (TD-Lambda, Q-learning, LSPE, LSTD, etc) - simulation and "least squares fit"
- Rollout algorithms (on-line)
  - Simulate the system under some (good heuristic) policy starting from the state of interest.
  - Use the cost of the heuristic (or a lower bound) as cost approximation

# SIMULATION AND LEARNING

- Simulation (learning by experience): used to compute the (approximate) cost-to-go is a key distinctive aspect of NDP
- Important advantage: A detailed model of the system not necessary - use a simulator instead
- In case of parametric approximation: off-line learning
- In case of a rollout algorithm: on-line learning is used (we learn only the cost values needed by on-line simulation)

# PARAMETRIC APPROXIMATION: CHESS PARADIGM

- Chess playing computer programs
- State = board position
- Score of position: “Important features” appropriately weighted



# TRAINING

- In chess: Weights are "hand-tuned"
- In more sophisticated methods: Weights are determined by using simulation-based training algorithms
- TD($\lambda$), Q-Learning, Least Squares Policy Evaluation (LSPE), Least Squares Temporal Differences (LSTD), extended Kalman filtering, etc
- All of these methods are based on DP ideas of policy iteration and value iteration

# POLICY IMPROVEMENT PRINCIPLE

- Given a current policy, define a new policy as follows:

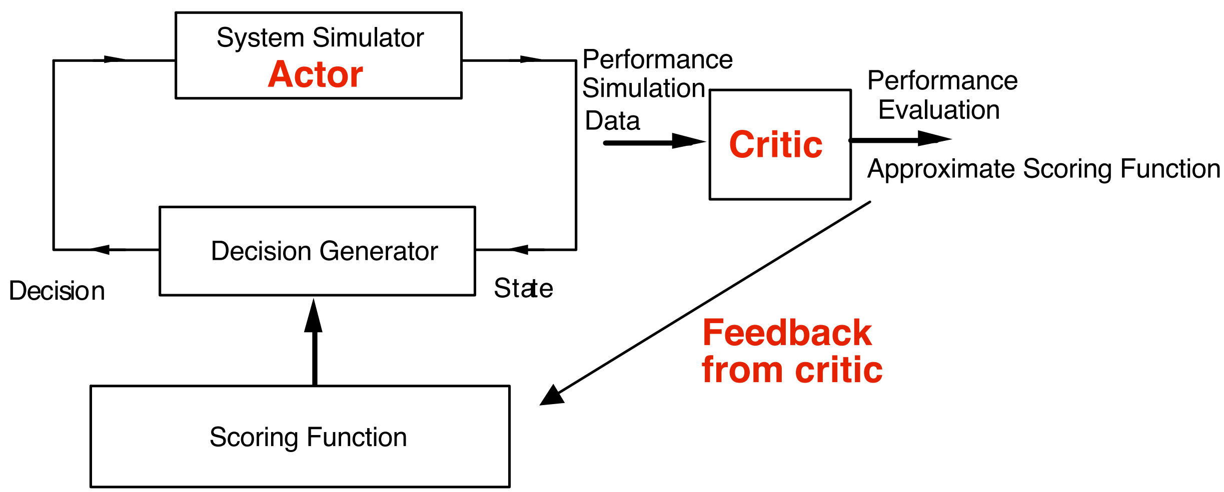
  At each state minimize

  Current stage cost + cost-to-go of current policy (starting from the next state)

- Policy improvement result: New policy has improved performance over current policy
- If the cost-to-go is approximate, the improvement is “approximate”
- Oscillation around the optimal; error bounds

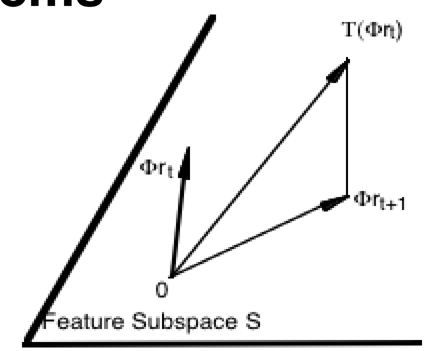
# ACTOR/CRITIC SYSTEMS

- Metaphor for policy improvement/evaluation
- **Actor implements** current policy
- **Critic evaluates** the performance; passes feedback to the actor
- Actor changes policy

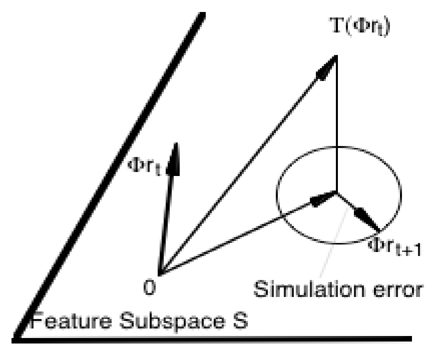


# POLICY EVALUATION BY VALUE ITERATION

- **Value iteration to evaluate the cost of a fixed policy:**
  **$J_{t+1} = T(J_t)$, where T is the DP mapping**
- **Value iteration with linear function approximation:**
  **$\Phi r_{t+1} = \Pi T(\Phi r_t)$**
  **where $\Phi$ is a matrix of basis functions/features and $\Pi$ is projection w/ respect to steady-state distribution norm**
- **Remarkable Fact: $\Pi T$ is a contraction for discounted and other problems**




Value Iteration with Linear Function Approximation



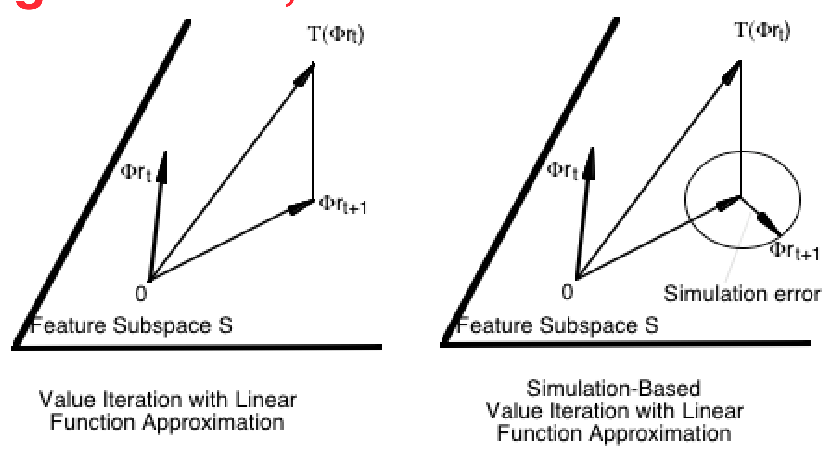

Simulation-Based Value Iteration with Linear Function Approximation

# LSPE: SIMULATION-BASED IMPLEMENTATION

- **Simulation-based implementation of $\Phi r_{t+1} = \Pi T(\Phi r_t)$ with an infinitely long trajectory, and least squares**

  **$\Phi r_{t+1} = \Pi T(\Phi r_t)$ + Diminishing simulation noise**

- **Interesting convergence theory (see papers at www site)**
- **Use of the steady-state distribution norm is critical**
- **Optimal convergence rate; much better than TD(lambda)**

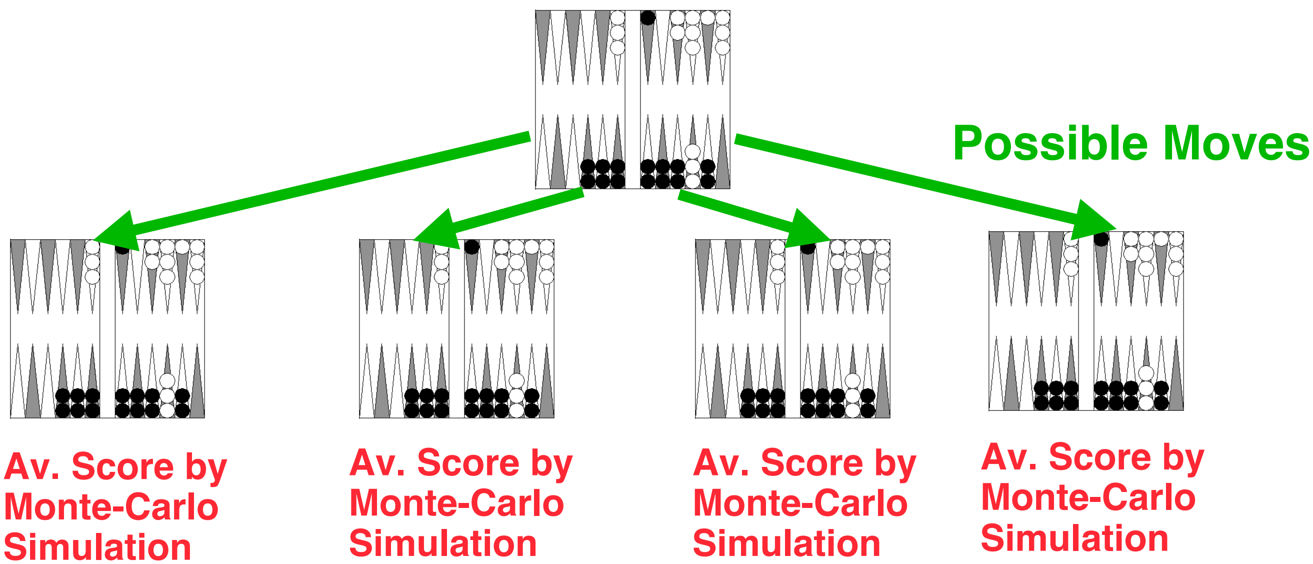


# SUMMARY OF ACTOR-CRITIC SYSTEMS

- A lot of mathematical analysis, insight, and practical experience are now available
- There is solid theory for:
  - Methods w/ exact (lookup table) cost representations
  - Policy evaluation methods with linear function aprpoximation [TD(lambda), LSPE, LSTD]
- In approximate policy iteration, typically, improved policies are obtained early, then the method oscillates
- On-line computation is small
- Training is challenging and time-consuming
- Less suitable when problem data changes frequently

# ROLLOUT POLICIES: BACKGAMMON PARADIGM

- On-line (approximate) cost-to-go calculation by simulation of some base policy (heuristic)
- Rollout: action w/ best simulation results
- Rollout is one-step policy iteration



# COST IMPROVEMENT PROPERTY

- Generic result: Rollout improves on Base
- A special case of policy iteration/policy improvement
- Extension to multiple base heuristics:
  - From each next state, run multiple heuristics
  - Use as value of the next state the best heuristic value
  - Cost improvement: The rollout algorithm performs at least as well as each of the base heuristics
- Interesting fact: The classical open-loop feedback control policy is a special case of rollout (base heuristic is the optimal open-loop policy)
- In practice, substantial improvements over the base heuristic(s) have been observed
- Major drawback: Extensive Monte-Carlo simulation

# STOCHASTIC PROBLEMS

- Major issue: Computational burden of Monte-Carlo simulation
- Motivation to use “approximate” Monte-Carlo
- Approximate Monte-Carlo by certainty equivalence: Assume future unknown quantities are fixed at some typical values
- Advantage : Single simulation run per next state, but some loss of optimality
- Extension to multiple scenarios (see Bertsekas and Castanon, 1997)

# ROLLOUT ALGORITHM PROPERTIES

- Forward looking (the heuristic runs to the end)
- Self-correcting (the heuristic is reapplied at each time step)
- Suitable for on-line use
- Suitable for replanning
- Suitable for situations where the problem data are a priori unknown
- Substantial positive experience with many types of optimization problems, including combinatorial (e.g., scheduling)

# DETERMINISTIC PROBLEMS

- ONLY ONE simulation trajectory needed
- Use heuristic(s) for approximate cost-to-go calculation
  - At each state, consider all possible next states, and run the heuristic(s) from each
  - Select the next state with best heuristic cost
- Straightforward to implement
- Cost improvement results are sharper (Bertsekas, Tsitsiklis, Wu, 1997, Bertsekas 2005)
- Extension to constrained problems

# MODEL PREDICTIVE CONTROL

- **Motivation: Deal with state/control constraints**
- **Basic MPC framework**
  - Deterministic discrete time system $x_{k+1} = f(x_k,u_k)$
  - Control contraint U, state constraint X
  - Quadratic cost per stage: x'Qx+u'Ru
- **MPC operation: At the typical state x**
  - Drive the state to 0 in m stages with minimum quadratic cost, while observing the constraints
  - Use the 1st component of the m-stage optimal control sequence, discard the rest
  - Repeat at the next state

# ADVANTAGES OF MPC

- It can deal explicitly with state and control constraints
- It can be implemented using standard deterministic optimal control methodology
- Key result: The resulting (suboptimal) closed-loop system is stable (under a "constrained controllability assumption" - Keerthi/Gilbert, 1988)
- Connection with infinite-time reachability
- Extension to problems with set-membership description of uncertainty

# CONNECTION OF MPC AND ROLLOUT

- MPC <==> Rollout with suitable base heuristic
- Heuristic: Apply the (m-1)-stage policy that drives the state to 0 with minimum cost
- Stability of MPC <==> Cost improvement of rollout
- Base heuristic stable ==> Rollout policy is also stable

# EXTENSIONS

- The relation with rollout suggests more general MPC schemes:
  - Nontraditional control and/or state constraints
  - Set-membership disturbances
- The success of MPC should encourage the use of rollout

# RESTRICTED STRUCTURE POLICIES

- General suboptimal control scheme
- At each time step: Impose restrictions on future information or control
- Optimize the future under these restrictions
- Use 1st component of the restricted policy
- Recompute at the next step

- Special cases:
  - Rollout, MPC: Restrictions on future control
  - Open-loop feedback control: Restrictions on future information
- Main result for the suboptimal policy so obtained:

It has better performance than the restricted policy

# CONCLUDING REMARKS

- NDP is a broadly applicable methodology; addresses optimization problems that are intractable in other ways
- Many off-line and on-line methods to choose from
- Interesting theory
- No need for a detailed model; a simulator suffices
- Computational requirements are substantial
- Successful application is an art
- Rollout has been the most consistently successful methodology
- Rollout has interesting connections with other successful methodologies such as MPC and open-loop feedback control