A fuzzy stopping problem with additive weighting function

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Abstract

A stopping problem for a dynamic fuzzy system is considered. We will formulate the problem which is concerned with fuzzy rewards and fuzzy stopping times. The objective is to adopt a notion of general utility, in case that a weighting function is additive, and construct an optimal fuzzy stopping time. The validity of the approach is proved by using α-cuts of fuzzy sets.

Keywords: Fuzzy stopping problem; dynamic fuzzy system; α-cuts of fuzzy sets; optimal fuzzy stopping time.

Some of topics which we will treat in this paper is cited as follows.

• Fuzzy decision:

Fuzzy decision making originated by Bellman/Zadeh (Mng. Sci. '70) is a multi-stage process in which the goals and/or the constraints are fuzzy. The dynamic programming method is shown to be a powerful technique for these problem by Bellman/Zadeh (Mng. Sci. '70) and Esogbue/Bellman (TIMS/Studies '84) . Also, for a wide application, it is desirable to develop a theory of multi-stage decision process under fuzzification of the state and its transition by fuzzy relation. Baldwin/Pilsworth (J. Math. Anal. Appl. '82) have proposed the multi-stage decision model described by fuzzy mappings which is an extension of the Bellman-Zadeh model. And they have taken an optimal decision which maximizes the measure of the truthness of "control and goal constraints satisfied".

• Markov-type FDP under the discount reward:

Markov-type fuzzy decision processes (FDP's, for short) with a bounded fuzzy reward are defined by Yoshida etc. (Fuz. Set. Sys. '93). We have developed its optimization under the discount reward criterion. Also, the long-run

average reward of some dynamic fuzzy system has been specified in our another paper Yoshida etc. (Fuz. Set. Sys. '94). However, the optimization was not considered there. So we will specify the long-run average fuzzy reward from a fuzzy policy and develop its optimization by the so-called "fuzzy max order" on the convex fuzzy numbers under the ergodicity (contraction) condition for the fuzzy state transition and the continuity condition for the fuzzy reward relation Kurano etc. (Eur. J. Opr. Res. '94).

For the average fuzzy reward case, a relative value function is considered in the fuzzy configration. The average reward from any admissible stationary policy is characterized as a unique solution of the associated equation. It will be useful in the policy improvement. Moreover, using the "vanishing discount factor" approach which is well-known in the theory of Markov decision processes, we derive the optimality equation under the average fuzzy reward criterion.

• g-utility and OLA stopping times:

The paper by Kadota etc. (Bul. Inf. Cyb. '96) is concerned with general utility treatment of optimal stopping problems for a denumerable Markov chain. And the validity of one-step look ahead policy(OLA policy) is discussed under general utility criteria. Moreover, the results are applied to the case of exponential utility functions and characterization of "risk-averse" or "risk-seeking" is given. In the present paper we will show how the problem of this paper is induced to use the technique, so called as Fuzzification.

• Fuzzy random variables and fuzzy stopping problem:

Fuzzy random variables was first studied by Puri/Ralescu (Ann. Prob. '85, J. Math. Anal. Appl. '86) and have been studied by many authors. Stojaković (Fuz. Set. Sys. '92) discussed fuzzy conditional expectation and Puri/Ralescu (J. Math. Anal. Appl. '91) studied fuzzy martingales. As the concrete strategy for the optimal stopping problem, the strategy of "One-step Look Ahead policy" is well known. Inducing the general utility function, Kurano etc. (preprint) and Kurano etc. (preprint) consider it in a fuzzy configuration.