or equivalently

$$v(x) = \begin{cases} \varphi(x), & x \in B, \\ \mathbb{N}_C[P_B\varphi](x), & x \in C, \end{cases} \tag{1.6}$$

if the One-step Look Ahead (abbreviated to OLA) rule is optimal, where $\mathbb{N} = \lim_{n\to\infty} \sum_{k=0}^{n} P^k$ is a potential operator and P_B, $\mathbb{N}_C$ mean the restriction of P and $\mathbb{N}$ on the set B or C, respectively.

The sets B and C are defined by the following (1.7), which are the regions where the decision is made either to stop or to continue, respectively. The OLA rule is that stops are made at the first time when the process enters a state in which stopping is at least as good as continuing for exactly one more period and then stopping. The rule is also well known as the monotone case of Markov sequence in [1]. To be precise, let

$$\begin{aligned} B &= \{x \in S; P\varphi(x) - \varphi(x) \le 0\}, \\ C &= \text{the complement of } B. \end{aligned} \tag{1.7}$$

The decision rule based on the stopping time ν_B, the first hitting time of the set B, is referred to frequently as the OLA rule, hereafter. If the rule is optimal for the problem, we shall say that the OLA rule is optimal. If the set B is closed, that is,

$$P(x, C) = 0, \qquad \text{for } x \in B, \tag{1.8}$$

and if it satisfies

$$\nu_B < \infty \text{ a.e. } P^x, \qquad X_0 = x \in S, \tag{1.9}$$

then the stopping time is optimal; that is, the OLA rule is optimal [4,5]. In [3], the explicit optimal value is obtained when the OLA rule is optimal, and it is applied to the best choice problem. Unlike the problem in which assumption (1.9) holds for all S, one must consider, in the game variant of the OLA rule, the case of $\nu_B = \infty$ on some set.

In this paper, our aim is to show an explicit expression for the value of zero-sum game variant, the so-called Dynkin's stopping game [6,7]. Furthermore, it is proved that the game value in this case is the sum of two independent maximal/minimal values with a zero reward at nonstopping for the standard stopping problems.

In Section 2, Dynkin's stopping game is considered when the OLA rule is optimal. The game value of the problem is expressed by using a potential operator and is decomposed as the sum of two independent maximal/minimal stopping problems. This is simpler than that of Bismut [8]. To discuss the standard stopping problem under an extended condition of the OLA rule, Section 3 considers it under the k-SLA rule, an abbreviation for the *k-Step* ($k \ge 1$) *Look Ahead* rule [9]. We shall express the optimal value of the standard problem under this rule. Again, Dynkin's stopping game is considered in Section 4. By taking k tend to infinity, the relation between the value of Dynkin's game and that of the standard stopping problem is obtained under the infinity-SLA rule.

2. DYNKIN'S STOPPING GAME UNDER THE OLA RULE

The formulation of Dynkin's stopping game is as follows. Two players I and II observe a Markov chain $\{(X_n, \mathcal{F}_n); n \ge 0\}$ with the stationary transition probability P on the countable state space S. Each of them chooses a stopping time adapted to $\{\mathcal{F}_n\}$ as one's strategy. If τ and σ are strategies of Player I and II, respectively, the payoff function is of the form

$$R(\tau, \sigma) = \varphi(X_\tau)\mathbf{1}_{\{\tau<\sigma\}} + \psi(X_\sigma)\mathbf{1}_{\{\tau>\sigma\}} + \chi(X_\tau)\mathbf{1}_{\{\tau=\sigma\}}, \tag{2.1}$$

where the function $\varphi(x)$, $\psi(x)$, and $\chi(x)$ on $x \in S$ are supposed to be given. Earlier stopping of players can stop the observation, and so $\tau \wedge \sigma = \min\{\tau, \sigma\}$ is the termination for the process. To avoid the nonterminated case, a pair of strategies

$$\tau \wedge \sigma < \infty$$