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Explicit Optimal Value for Dynkin's Stopping Game

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Abstract—Under the One-step Look Ahead rule of Dynamic Programming, an explicit game value of Dynkin's stopping problem for a Markov chain is obtained by using a potential operator. The condition on the *One-step rule* could be extended to the k-step and infinity-step rule. We shall also decompose the game value as the sum of two explicit functions under these rules.

Keywords—Optimal stopping problem, Dynkin's game, One-step Look Ahead rule, Markov potential theory.

1. INTRODUCTION AND SUMMARY

Let $\{(X_n, \mathcal{F}_n); n \geq 0\}$ be a Markov chain with a countable state space S having stationary transition probabilities $P(x, A)$, $x \in S$. Suppose a function $\varphi(x)$, $x \in S$ is given. The standard optimal stopping problem is to find a stopping time which maximizes $E^x[\varphi(X_\tau)] = \mathbf{E}[\varphi(X_\tau) \mid X_0 = x]$ in the class of all finite stopping times τ adapted to $\{\mathcal{F}_n; n \geq 0\}$. The optimal value is denoted by

$$v(x) = \sup_{0 \leq \tau < \infty} E^x[\varphi(X_\tau)], \qquad x \in S. \tag{1.1}$$

The detailed analyses are discussed by many authors such as Chow, Robbins and Siegmund [1], Shiryaev [2], etc. By the Dynamic Programming method, the optimality equation becomes

$$v(x) = \max \left\{ \begin{matrix} \text{stop} & \text{conti.} \\ \varphi(x), & Pv(x) \end{matrix} \right\}, \qquad x \in S, \tag{1.2}$$

where $Pv(x) = \sum_{y \in S} v(y) P(x, y)$. We shall rewrite this equation as

$$v(x) = \begin{cases} \varphi(x) & \text{on } \{x \in S;\ \varphi(x) \geq Pv(x)\}, \\ Pv(x) & \text{on } \{x \in S;\ \varphi(x) < Pv(x)\}, \end{cases} \tag{1.3}$$

or

$$v(x) - Pv(x) = (\varphi - Pv)^+(x), \qquad x \in S \tag{1.4}$$

in comparison with the game variant of the problem in the later section. Hereafter, we shall use the superscript $\pm$ as $a^+ = \max(a, 0)$, $a^- = -\min(a, 0)$.

In the previous paper [3], we obtained the explicit expression of the optimal value as

$$v(x) = \varphi(x) + \mathbb{N}(P\varphi - \varphi)^+(x), \qquad x \in S, \tag{1.5}$$

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