

PIVOTAL METHOD

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The summary of three operations: Fxx, Gxx, Hxx
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Given matrix MA :

-> MA:matrix([a_1,a_2,a_3],[b_1,b_2,b_3],[c_1,c_2,c_3]);

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \quad (\% \text{ o5})$$

-> For "row" operation, multiply the matrix by "left" !!
For "column" operation, multiply the matrix by "right" !!

(1-1) F11 : r1 \to \from r2; Exchange r1 with r2.

-> F11:matrix([0,1,0],[1,0,0],[0,0,1]);

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\% \text{ o1})$$

-> F11.MA;

$$\begin{pmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \quad (\% \text{ o6})$$

(1-2) F12: r2 \to \from r3; Exchange r2 with r3.

-> F12:matrix([1,0,0],[0,0,1],[0,1,0]);

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (\% \text{ o19})$$

-> F12.MA;

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \end{pmatrix} \quad (\% \text{ o20})$$

(2) G11: $r_2 * c$; Multiply r_2 by k .

$\rightarrow$ `G11:matrix([1,0,0],[0,k,0],[0,0,1]);`

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\% \text{ o15})$$

$\rightarrow$ `G11.MA;`

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 k & b_2 k & b_3 k \\ c_1 & c_2 & c_3 \end{pmatrix} \quad (\% \text{ o16})$$

(2-1) G12: $r_3 * k$; Multiply r_3 by k .

$\rightarrow$ `G12:matrix([1,0,0],[0,1,0],[0,0,k]);`

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & k \end{pmatrix} \quad (\% \text{ o17})$$

$\rightarrow$ `G12.MA;`

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 k & c_2 k & c_3 k \end{pmatrix} \quad (\% \text{ o18})$$

(3-1) H11: $r_1 * k + r_2$; Multiply r_1 by k add r_2

$\rightarrow$ `H11:matrix([1,0,0],[k,1,0],[0,0,1]);`

$$\begin{pmatrix} 1 & 0 & 0 \\ k & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\% \text{ o4})$$

$\rightarrow$ `H11.MA;`

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ a_1 k + b_1 & a_2 k + b_2 & a_3 k + b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \quad (\% \text{ o8})$$

(3-2) H12: $r_1 \cdot k + r_3$; Multiply r_1 by k add r_3

$\rightarrow$ `H12:matrix([1,0,0],[0,1,0],[k,0,1]);`

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ k & 0 & 1 \end{pmatrix} \quad (\% \text{ o9})$$

$\rightarrow$ `H12.MA;`

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ a_1k + c_1 & a_2k + c_2 & a_3k + c_3 \end{pmatrix} \quad (\% \text{ o10})$$

(3-3) H13: $r_2 \cdot k + r_3$; Multiply r_2 by k add r_3

$\rightarrow$ `H13:matrix([1,0,0],[0,1,0],[0,k,1]);`

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & k & 1 \end{pmatrix} \quad (\% \text{ o11})$$

$\rightarrow$ `H13.MA;`

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ b_1k + c_1 & b_2k + c_2 & b_3k + c_3 \end{pmatrix} \quad (\% \text{ o12})$$

(3-4) H14: $r_2 \cdot k + r_1$; Multiply r_2 by k add r_1

$\rightarrow$ `H14:matrix([1,k,0],[0,1,0],[0,0,1]);`

$$\begin{pmatrix} 1 & k & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\% \text{ o13})$$

$\rightarrow$ `H14.MA;`

$$\begin{pmatrix} b_1k + a_1 & b_2k + a_2 & b_3k + a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \quad (\% \text{ o14})$$

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Now, the pivote process begins as follows;

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Step 1;

(% i2) A:matrix([2,-2,1],[1,2,-4],[-3,2,0]);

$$\begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & -4 \\ -3 & 2 & 0 \end{pmatrix} \quad (\% \text{ o2})$$

(% i3) F1:matrix([0,1,0],[1,0,0],[0,0,1]);

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\% \text{ o3})$$

-> F1.A;

$$\begin{pmatrix} 1 & 2 & -4 \\ 2 & -2 & 1 \\ -3 & 2 & 0 \end{pmatrix} \quad (\% \text{ o8})$$

-> A2:F1.A;

$$\begin{pmatrix} 1 & 2 & -4 \\ 2 & -2 & 1 \\ -3 & 2 & 0 \end{pmatrix} \quad (\% \text{ o9})$$

H1: r1*(-2) + r2

(% i4) H1: matrix([1,0,0],[-2,1,0],[0,0,1]);

$$\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\% \text{ o4})$$

-> A3:H1.A2;

$$\begin{pmatrix} 1 & 2 & -4 \\ 0 & -6 & 9 \\ -3 & 2 & 0 \end{pmatrix} \quad (\% \text{ o12})$$

H2 : r1 *3 + r3;

(% i5) H2:matrix([1,0,0],[0,1,0],[3,0,1]);

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} \quad (\% \text{ o5})$$

-> A3:H2.A3;

$$\begin{pmatrix} 1 & 2 & -4 \\ 0 & -6 & 9 \\ 0 & 8 & -12 \end{pmatrix} \quad (\% \text{ o15})$$

Step 2;

G1 : r2*(-1/6);

(% i6) G1:matrix([1,0,0],[0,-1/6,0],[0,0,1]);

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{6} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\% \text{ o6})$$

-> A4:G1.A3;

$$\begin{pmatrix} 1 & 2 & -4 \\ 0 & 1 & -\frac{3}{2} \\ 0 & 8 & -12 \end{pmatrix} \quad (\% \text{ o17})$$

H3: r2*(-8) + r3;

(% i9) H3:matrix([1,0,0],[0,1,0],[0,-8,1]);

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -8 & 1 \end{pmatrix} \quad (\% \text{ o9})$$

-> A5:H3.A4;

$$\begin{pmatrix} 1 & 2 & -4 \\ 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 0 \end{pmatrix} \quad (\% \text{ o19})$$

H4 : r2*(-2) + r1;

(% i7) H4:matrix([1,-2,0],[0,1,0],[0,0,1]);

$$\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\% \text{ o7})$$

-> A6: H4.A5;

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 0 \end{pmatrix} \quad (\% \text{ o21})$$

These end the procedure. =====

rank of calcuration:

-> A;

$$\begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & -4 \\ -3 & 2 & 0 \end{pmatrix} \quad (\% \text{ o21})$$

-> rank(A);

$$2 \quad (\% \text{ o22})$$

Problem : Find Q such as

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$$Q.A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

(Ans) Q = H4.H3.G1.H2.H1.F1;

$$Q.A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

(% i21) Q:H4.H3.G1.H2.H1.F1;

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ -\frac{1}{6} & \frac{1}{3} & 0 \\ \frac{4}{3} & \frac{1}{3} & 1 \end{pmatrix} \quad (\% \text{ o21})$$

(% i11) QA:Q.A;

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 0 \end{pmatrix} \quad (\% \text{ o11})$$

columns operation:

(% i12) Hr1:matrix([1,0,1],[0,1,0],[0,0,1]);

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\% \text{ o12})$$

(% i13) QA.Hr1;

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 0 \end{pmatrix} \quad (\% \text{ o13})$$

(% i14) Hr2:matrix([1,0,0],[0,1,3/2],[0,0,1]);

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 1 \end{pmatrix} \quad (\% \text{ o14})$$

(% i22) P:Hr1.Hr2;

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 1 \end{pmatrix} \quad (\% \text{ o22})$$

(% i18) A;

$$\begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & -4 \\ -3 & 2 & 0 \end{pmatrix} \quad (\% \text{ o18})$$

(% i23) Q.A.P;

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (\% \text{ o23})$$