

## 数学演習 (1) 第 13 回 定積分 解答

I.

$$(1) \int_{-1}^2 f(x) dx = \int_{-1}^0 (3x^2 + 7x) dx + \int_0^1 3\sqrt{x} dx + \int_1^2 \frac{1}{x^2} dx = 0$$

$$(2) \int_0^2 (2x - 3)^3 dx = -10$$

$$(3) \int_0^1 xe^{-x^2} dx = \frac{1}{2}(1 - \frac{1}{e})$$

$$(4) \int_e^{e^2} \frac{\log x}{x} dx = \frac{3}{2}$$

$$(5) \int_0^{1/2} \frac{x}{\sqrt{1-x^2}} dx = 1 - \frac{\sqrt{3}}{2}$$

$$(6) \int_0^\pi \frac{\sin x}{1 + \cos^2 x} dx = \frac{\pi}{2}$$

$$(7) \int_0^{\pi/4} \tan x dx = \frac{1}{2} \log 2$$

$$(8) \int_0^\pi x \sin 2x dx = -\frac{\pi}{2}$$

$$(9) \int_1^e x \log x dx = \frac{1}{4}(e^2 + 1)$$

$$(10) \int_0^1 xe^{-x} dx = 1 - \frac{2}{e}$$

$$(11) \int_0^1 x^2 e^{-x} dx = 2 - \frac{5}{e}$$

II.

$$(1) \int_0^{2\pi} \sin mx \cos nx dx = 0$$

$$(2) \int_0^{2\pi} \sin mx \sin nx dx = \begin{cases} 0 & (m \neq n) \\ \pi & (m = n) \end{cases}$$

$$(3) \int_0^{2\pi} \cos mx \cos nx dx = \begin{cases} 0 & (m \neq n) \\ \pi & (m = n) \end{cases}$$

$(\sin \alpha \cos \beta = \frac{1}{2} \{ \sin(\alpha - \beta) + \sin(\alpha + \beta) \}, \cos \alpha \cos \beta = \frac{1}{2} \{ \cos(\alpha - \beta) + \cos(\alpha + \beta) \}, \sin \alpha \sin \beta = \frac{1}{2} \{ \cos(\alpha - \beta) - \cos(\alpha + \beta) \},$  特に  $\alpha = \beta$  のとき

$$\cos^2 \alpha = \frac{1}{2}(1 + \cos 2\alpha), \quad \sin^2 \alpha = \frac{1}{2}(1 - \cos 2\alpha)$$