数学演習 (1)　第 3 回　関数の極限　解答

I.

x	-4	-3	-2	-1	0	1	2	3	4
2^x	$1/16$	$1/8$	$1/4$	$1/2$	1	2	4	8	16
2^{-x}	16	8	4	2	1	$1/2$	$1/4$	$1/8$	$1/16$

x	$1/32$	$1/16$	$1/8$	$1/4$	$1/2$	1	$\sqrt{2}$	2	4	8
$\log_2 x$	-5	-4	-3	-2	-1	0	$1/2$	1	2	3

II. 略

III.

(1) $\displaystyle\lim_{x\to\infty}\log x = +\infty,\quad \lim_{x\to+0}\log x = -\infty$

(2) $\displaystyle\lim_{x\to\infty} e^{1/x} = 1,\quad \lim_{x\to-\infty} e^{1/x} = 1,\quad \lim_{x\to+0} e^{1/x} = +\infty,\quad \lim_{x\to-0} e^{1/x} = 0$

(3) $\displaystyle\lim_{x\to1}\frac{x^2-2x}{3x^2-7x+2} = \frac{1}{2},\quad \lim_{x\to2}\frac{x^2-2x}{3x^2-7x+2} = \lim_{x\to2}\frac{x(x-2)}{(3x-1)(x-2)} = \lim_{x\to2}\frac{x}{3x-1} = \frac{2}{5},$

$\displaystyle\lim_{x\to\infty}\frac{x^2-2x}{3x^2-7x+2} = \lim_{x\to\infty}\frac{1-2/x}{3-7/x+2/x^2} = \frac{1}{3}$

(4) $\displaystyle\lim_{x\to\infty}\frac{e^x-e^{-x}}{e^x+e^{-x}} = \lim_{x\to\infty}\frac{1-e^{-2x}}{1+e^{-2x}} = 1,\quad \lim_{x\to-\infty}\frac{e^x-e^{-x}}{e^x+e^{-x}} = \lim_{x\to-\infty}\frac{e^{2x}-1}{e^{2x}+1} = -1$

(5) $\displaystyle\lim_{x\to1}\frac{1-\log x}{1+\log x} = 1,\ \lim_{x\to\infty}\frac{1-\log x}{1+\log x} = \lim_{x\to\infty}\frac{(\log x)^{-1}-1}{(\log x)^{-1}+1} = -1,\ \lim_{x\to+0}\frac{1-\log x}{1+\log x} = \lim_{x\to+0}\frac{(\log x)^{-1}-1}{(\log x)^{-1}+1} = -1$

(6) $\displaystyle\lim_{x\to0}\frac{\sin 6x}{2x} = 3$

(7) $\displaystyle\lim_{x\to\infty}\left(1+\frac{4}{x}\right)^{3x} = e^{12}$

(8) $\displaystyle\lim_{x\to0}(1-3x)^{2/x} = e^{-6}$

(9) $\displaystyle\lim_{x\to0}\frac{\log(1+6x)}{x} = 6$

(10) $\displaystyle\lim_{x\to0}\frac{e^{-8x}-1}{4x} = -2$

(11) $\displaystyle\lim_{x\to0}\frac{\arcsin x}{x} = \lim_{y\to0}\frac{y}{\sin y} = 1$

(12) $\displaystyle\lim_{x\to\infty}\arctan x = \frac{\pi}{2},\quad \lim_{x\to-\infty}\arctan x = -\frac{\pi}{2}$

(13) $\displaystyle\lim_{x\to\infty}(\sqrt{2x+1}-\sqrt{2x-1}) = \lim_{x\to\infty}\frac{(\sqrt{2x+1}-\sqrt{2x-1})(\sqrt{2x+1}+\sqrt{2x-1})}{\sqrt{2x+1}+\sqrt{2x-1}} = \lim_{x\to\infty}\frac{2}{\sqrt{2x+1}+\sqrt{2x-1}} = 0$

(14) $\displaystyle\lim_{x\to\infty}\sqrt{x}\,(\sqrt{2x+1}-\sqrt{2x-1}) = \lim_{x\to\infty}\frac{2\sqrt{x}}{\sqrt{2x+1}+\sqrt{2x-1}} = \lim_{x\to\infty}\frac{2}{\sqrt{2+1/x}+\sqrt{2-1/x}} = \frac{1}{\sqrt{2}}$