2019.5.31

数学演習 (1)　第 6 回　高次導関数　解答

I. (1) $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}(ax+b)^c = \dfrac{\mathrm{d}}{\mathrm{d}x}ac(ax+b)^{c-1} = a^2c(c-1)(ax+b)^{c-2}$

(2) $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\,x^2\log x = \dfrac{\mathrm{d}}{\mathrm{d}x}(2x\log x + x) = 2\log x + 3$

(3) $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\log(ax+b) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{a}{ax+b} = \dfrac{-a^2}{(ax+b)^2}$

(4) $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\dfrac{1}{x^2+1} = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{-2x}{(x^2+1)^2} = \dfrac{-2(x^2+1)^2 - (-2x)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4} = \dfrac{-2(x^2+1)+8x^2}{(x^2+1)^3} = \dfrac{6x^2-2}{(x^2+1)^3}$

(5) $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\,e^{\sin x} = \dfrac{\mathrm{d}}{\mathrm{d}x}(e^{\sin x}\cos x) = (\cos^2 x - \sin x)e^{\sin x}$

(6) $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\tan ax = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{a}{\cos^2 ax} = \dfrac{2a^2\sin ax}{\cos^3 ax}$,　或いは $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\tan ax = \dfrac{\mathrm{d}}{\mathrm{d}x}a(1+\tan^2 ax) = 2a^2(1+\tan^2 ax)\tan ax$

(7) $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\arcsin ax = \dfrac{\mathrm{d}}{\mathrm{d}x}a(1-a^2x^2)^{-1/2} = \dfrac{a^3x}{(1-a^2x^2)^{3/2}}$

(8) $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\arctan ax = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{a}{1+a^2x^2} = \dfrac{-2a^3x}{(1+a^2x^2)^2}$

(9) $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\sqrt{x^2+1} = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{x}{\sqrt{x^2+1}} = \dfrac{\sqrt{x^2+1} - x\cdot\frac{x}{\sqrt{x^2+1}}}{x^2+1} = \dfrac{1}{(x^2+1)\sqrt{x^2+1}}$

II. (1) 略　(2) $f^{(n)}(x) = \dfrac{n!}{(1-x)^{n+1}}$　$(n \geq 1)$

III. (1) $\dfrac{\mathrm{d}^n}{\mathrm{d}x^n}e^{ax} = a^n e^{ax}$

(2) $\dfrac{\mathrm{d}^n}{\mathrm{d}x^n}\sin ax = a^n\sin(ax + \dfrac{n\pi}{2})$, または m を 0 以上の整数として

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n}\sin ax = \begin{cases} a^n\sin ax & (n = 4m \text{ のとき}) \\ a^n\cos ax & (n = 4m+1 \text{ のとき}) \\ -a^n\sin ax & (n = 4m+2 \text{ のとき}) \\ -a^n\cos ax & (n = 4m+3 \text{ のとき}) \end{cases}$$

または m を 0 以上の整数として

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n}\sin ax = \begin{cases} (-1)^m a^{2m}\sin ax & (n = 2m \text{ のとき}) \\ (-1)^m a^{2m+1}\cos ax & (n = 2m+1 \text{ のとき}) \end{cases}$$

(3) $\dfrac{\mathrm{d}^n}{\mathrm{d}x^n}\cos ax = a^n\cos(ax + \dfrac{n\pi}{2})$, または m を 0 以上の整数として

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n}\cos ax = \begin{cases} a^n\cos ax & (n = 4m \text{ のとき}) \\ -a^n\sin ax & (n = 4m+1 \text{ のとき}) \\ -a^n\cos ax & (n = 4m+2 \text{ のとき}) \\ a^n\sin ax & (n = 4m+3 \text{ のとき}) \end{cases}$$

または m を 0 以上の整数として

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n}\cos ax = \begin{cases} (-1)^m a^{2m}\cos ax & (n = 2m \text{ のとき}) \\ (-1)^{m+1} a^{2m+1}\sin ax & (n = 2m+1 \text{ のとき}) \end{cases}$$

IV. $p = \dfrac{f''(\alpha)}{2}$, $q = f'(\alpha)$, $r = f(\alpha)$

V. $p = \dfrac{f'''(\alpha)}{3!}$, $q = \dfrac{f''(\alpha)}{2!}$, $r = f'(\alpha)$, $s = f(\alpha)$

VI. (1) $f(x) = 2(x-3)^2 + 17(x-3) + 39$
(2) $f(x) = 3(x-2)^2 + 11(x-2) + 12$
(3) $f(x) = 2(x-1)^3 + 12(x-1)^2 + 22(x-1) + 17$
(4) $f(x) = (x+4)^3 - 12(x+4)^2 + 48(x+4) - 64$