2018.7.13

数学演習 (1)　第 12 回　積分の計算　解答

I.

(1) $\displaystyle\int \frac{1}{1-x}\,dx = -\log|1-x| + C$

(2) $\displaystyle\int \frac{1}{(4x+1)^2}\,dx = -\frac{1}{4(4x+1)} + C$

(3) $\displaystyle\int \frac{3x+7}{(x-3)(x+1)}\,dx = \int\left(\frac{4}{x-3} - \frac{1}{x+1}\right)dx = 4\log|x-3| - \log|x+1| + C$

(4) $\displaystyle\int \frac{1}{4x^2-1}\,dx = \frac{1}{2}\left(\int \frac{1}{2x-1}\,dx - \int \frac{1}{2x+1}\,dx\right) = \frac{1}{4}\log\left|\frac{2x-1}{2x+1}\right| + C$

(5) $\displaystyle\int \frac{4}{x^2(x+2)}\,dx = \int\left(-\frac{1}{x} + \frac{2}{x^2} + \frac{1}{x+2}\right)dx = \log\left|\frac{x+2}{x}\right| - \frac{2}{x} + C$

(6) $\displaystyle\int \frac{x}{(x-2)(x-1)^2}\,dx = \int\left(\frac{2}{x-2} - \frac{2}{x-1} - \frac{1}{(x-1)^2}\right)dx = 2\log\left|\frac{x-2}{x-1}\right| + \frac{1}{x-1} + C$

(7) $\displaystyle\int \frac{1}{x^2+6}\,dx = \frac{1}{\sqrt{6}}\arctan\frac{x}{\sqrt{6}} + C$

(8) $\displaystyle\int \frac{x-4}{x^2+6}\,dx = \frac{1}{2}\log(x^2+6) - \frac{4}{\sqrt{6}}\arctan\frac{x}{\sqrt{6}} + C$

(9) $\displaystyle\int \frac{1}{x^2+2x+4}\,dx = \frac{1}{\sqrt{3}}\arctan\frac{x+1}{\sqrt{3}} + C$

(10) $\displaystyle\int \frac{x+2}{x^2+6x+10}\,dx = \frac{1}{2}\log(x^2+6x+10) - \arctan(x+3) + C$

(11) $\displaystyle\int \frac{2}{(x-1)(x^2+1)}\,dx = \int\left(\frac{1}{x-1} - \frac{x+1}{x^2+1}\right)dx = \log|x-1| - \frac{1}{2}\log(x^2+1) - \arctan x + C$

(12) $\displaystyle\int \frac{1}{x^3-1}\,dx = \int \frac{1}{3}\left(\frac{1}{x-1} - \frac{x+2}{x^2+x+1}\right)dx = \frac{1}{3}\log|x-1| - \frac{1}{6}\log(x^2-x+1) - \frac{1}{\sqrt{3}}\arctan\frac{2x+1}{\sqrt{3}} + C$