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数学演習 (1)　第 3 回　関数の極限　解答

I.

x	-4	-3	-2	-1	0	1	2	3	4
2^x	$1/16$	$1/8$	$1/4$	$1/2$	1	2	4	8	16
2^{-x}	16	8	4	2	1	$1/2$	$1/4$	$1/8$	$1/16$

x	$1/32$	$1/16$	$1/8$	$1/4$	$1/2$	1	$\sqrt{2}$	2	4	8
$\log_2 x$	-5	-4	-3	-2	-1	0	$1/2$	1	2	3

II. 略

III.

(1) $\displaystyle\lim_{x\to\infty}\log x=+\infty,\quad \lim_{x\to+0}\log x=-\infty$

(2) $\displaystyle\lim_{x\to\infty}e^{1/x}=1,\quad \lim_{x\to-\infty}e^{1/x}=1,\quad \lim_{x\to+0}e^{1/x}=+\infty,\quad \lim_{x\to-0}e^{1/x}=0$

(3) $\displaystyle\lim_{x\to1}\frac{x^2-2x}{2x^2-5x+2}=1,\quad \lim_{x\to2}\frac{x^2-2x}{2x^2-5x+2}=\lim_{x\to2}\frac{x(x-2)}{(2x-1)(x-2)}=\lim_{x\to2}\frac{x}{2x-1}=\frac{2}{3},$

$\displaystyle\lim_{x\to\infty}\frac{x^2-2x}{2x^2-5x+2}=\lim_{x\to\infty}\frac{1-2/x}{2-5/x+2/x^2}=\frac{1}{2}$

(4) $\displaystyle\lim_{x\to\infty}\frac{e^x-e^{-x}}{e^x+e^{-x}}=\lim_{x\to\infty}\frac{1-e^{-2x}}{1+e^{-2x}}=1,\quad \lim_{x\to-\infty}\frac{e^x-e^{-x}}{e^x+e^{-x}}=\lim_{x\to-\infty}\frac{e^{2x}-1}{e^{2x}+1}=-1$

(5) $\displaystyle\lim_{x\to1}\frac{1-\log x}{1+\log x}=1,\ \lim_{x\to\infty}\frac{1-\log x}{1+\log x}=\lim_{x\to\infty}\frac{(\log x)^{-1}-1}{(\log x)^{-1}+1}=-1,\ \lim_{x\to+0}\frac{1-\log x}{1+\log x}=\lim_{x\to+0}\frac{(\log x)^{-1}-1}{(\log x)^{-1}+1}=-1$

(6) $\displaystyle\lim_{x\to0}\frac{\sin 5x}{3x}=\frac{5}{3}$

(7) $\displaystyle\lim_{x\to\infty}\left(1+\frac{4}{x}\right)^{2x}=e^8$

(8) $\displaystyle\lim_{x\to0}(1-3x)^{2/x}=e^{-6}$

(9) $\displaystyle\lim_{x\to0}\frac{\log(1+5x)}{x}=5$

(10) $\displaystyle\lim_{x\to0}\frac{e^{-6x}-1}{7x}=-\frac{6}{7}$

(11) $\displaystyle\lim_{x\to0}\frac{\arcsin x}{x}=\lim_{y\to0}\frac{y}{\sin y}=1$

(12) $\displaystyle\lim_{x\to\infty}\arctan x=\frac{\pi}{2},\quad \lim_{x\to-\infty}\arctan x=-\frac{\pi}{2}$

(13) $\displaystyle\sqrt{2x+1}\,(\sqrt{x+1}-\sqrt{x})=\sqrt{2x+1}\,(\sqrt{x+1}-\sqrt{x})\frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}=\frac{\sqrt{2x+1}}{\sqrt{x+1}+\sqrt{x}}$ より

$$\lim_{x\to\infty}\sqrt{2x+1}\,(\sqrt{x+1}-\sqrt{x})=\lim_{x\to\infty}\frac{\sqrt{2x+1}}{\sqrt{x+1}+\sqrt{x}}=\frac{\sqrt{2}}{2}$$

(14) $\displaystyle\lim_{h\to0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\lim_{h\to0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$

(15) $\displaystyle\lim_{h\to0}\frac{\log(x+h)-\log x}{h}=\lim_{h\to0}\frac{\log(1+h/x)}{h}=\frac{1}{x}$

(16) $\displaystyle\lim_{h\to0}\frac{\sin(x+h)-\sin x}{h}=\lim_{h\to0}\frac{\sin h/2}{h/2}\cdot\cos\left(x+\frac{h}{2}\right)=\cos x$