

数学演習 (1) 第 13 回 定積分 解答

I.

$$(1) \int_{-1}^2 f(x) dx = \int_{-1}^0 \left(4x^3 + \frac{x}{3}\right) dx + \int_0^1 \sqrt{x} dx + \int_1^2 \frac{1}{x^2} dx = 0$$

$$(2) \int_{-1}^1 e^{ax} dx = \frac{e^a - e^{-a}}{a}$$

$$(3) \int_0^1 (3x-2)^3 dx = -\frac{5}{4}$$

$$(4) \int_0^\pi \frac{\sin x}{1 + \cos^2 x} dx = \frac{\pi}{2}$$

$$(5) \int_e^{e^2} \frac{1}{x \log x} dx = \log 2$$

$$(6) \int_0^1 \frac{x}{\sqrt{2-x}} dx = \frac{8}{3}\sqrt{2} - \frac{10}{3}$$

$$(7) \int_0^{\pi/4} \tan x dx = \frac{1}{2} \log 2$$

$$(8) \int_0^{\pi/6} x \sin 3x dx = \frac{1}{9}$$

$$(9) \int_0^\pi x \log x dx = \frac{e^2}{4} + 1$$

$$(10) \int_0^1 \arctan x dx = \frac{\pi}{4} - \frac{1}{2} \log 2$$

$$(11) \int_0^1 x^2 e^{-x} dx = 2 - \frac{5}{e}$$

II.

$$(1) \int_0^{2\pi} \sin mx \cos nx dx = 0$$

$$(2) \int_0^{2\pi} \sin mx \sin nx dx = \begin{cases} 0 & (m \neq n) \\ \pi & (m = n) \end{cases}$$

$$(3) \int_0^{2\pi} \cos mx \cos nx dx = \begin{cases} 0 & (m \neq n) \\ \pi & (m = n) \end{cases}$$

$(\sin \alpha \cos \beta = \frac{1}{2} \{\sin(\alpha - \beta) + \sin(\alpha + \beta)\}, \cos \alpha \cos \beta = \frac{1}{2} \{\cos(\alpha - \beta) + \cos(\alpha + \beta)\}, \sin \alpha \sin \beta = \frac{1}{2} \{\cos(\alpha - \beta) - \cos(\alpha + \beta)\})$, 特に $\alpha = \beta$ のとき

$$\cos^2 \alpha = \frac{1}{2}(1 + \cos 2\alpha), \quad \sin^2 \alpha = \frac{1}{2}(1 - \cos 2\alpha)$$

III. 略