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数学演習 (1)　第 1 回　数列の極限

解説：数列の収束と発散, 無限級数とその和

例　$\displaystyle\sum_{n=1}^{\infty} a^{n-1} = \frac{1}{1-a} \quad (|a| < 1)$

問題+宿題

I. 次の式で定まる数列 $\{a_n\}$ の極限を求めよ. (発散することもある)

(1) $a_n = 3^n$

(2) $a_n = -3^n$

(3) $a_n = (-3)^n$

(4) $a_n = -3^{-n}$

(5) $a_n = (-3)^{-n}$

(6) $a_n = 1 + 2\left(\dfrac{4}{5}\right)^n$

(7) $a_n = \sin\dfrac{n\pi}{2}$

(8) $a_n = \dfrac{8}{2n-1}$

(9) $a_n = \dfrac{6n^2+7n+4}{3n^2+n}$

(10) $a_n = \dfrac{2^{n+1}+1}{6^n+1}$

(11) $a_n = \dfrac{3^n-4^{n+1}}{4^n+(-3)^{n+1}}$

(12) $a_n = \dfrac{\sqrt{n}}{\sqrt{2n+1}+\sqrt{2n-1}}$

(13) $a_n = \sqrt{n^2+n} - n$

(14) $a_n = (-2)^n - \dfrac{4^n}{(-2)^n+1}$

(15) $a_n = 0.\underbrace{11\cdots1}_{n\text{ 個}}$

II. 次の無限級数 $\displaystyle\sum_{n=1}^{\infty} a_n$ について, 部分和 $\displaystyle S_n = \sum_{k=1}^{n} a_k = a_1 + a_2 + \cdots + a_n$ $(n \geq 1)$ を求めよ. さらに, 級数の和 $\displaystyle\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n = S$ を求めよ.

(1) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$　　(hint: $\dfrac{1}{n+1} - \dfrac{1}{n+2} =?$)

(2) $\displaystyle\sum_{n=1}^{\infty} \log\left(1 - \frac{1}{n+1}\right)$　　(hint: 級数は発散する)

(3) $\displaystyle\sum_{n=1}^{\infty} \log\left(1 - \frac{1}{(n+1)^2}\right)$