

付録 公式集

三角関数の公式

(1) 基本公式

$$\sin^2 \theta + \cos^2 \theta = 1, \quad 1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}, \quad 1 + \frac{1}{\tan^2 \theta} = \frac{1}{\sin^2 \theta}$$

(2) 加法定理

$$\begin{aligned}\sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \\ \tan(\alpha \pm \beta) &= \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \quad (\text{以上, 複号同順})\end{aligned}$$

(3) 2倍角の公式

$$\begin{aligned}\sin 2\alpha &= 2 \sin \alpha \cos \alpha \\ \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha \\ \tan 2\alpha &= \frac{2 \tan \alpha}{1 - \tan^2 \alpha}\end{aligned}$$

(4) 3倍角の公式

$$\begin{aligned}\sin 3\alpha &= 3 \sin \alpha - 4 \sin^3 \alpha, \quad \cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha \\ \tan 3\alpha &= \frac{3 \tan \alpha - \tan^3 \alpha}{1 - 3 \tan^2 \alpha}\end{aligned}$$

(5) 積を和, 差になおす公式

$$\begin{aligned}\sin \alpha \cos \beta &= \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{2} \\ \cos \alpha \sin \beta &= \frac{\sin(\alpha + \beta) - \sin(\alpha - \beta)}{2} \\ \cos \alpha \cos \beta &= \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{2} \\ \sin \alpha \sin \beta &= -\frac{\cos(\alpha + \beta) - \cos(\alpha - \beta)}{2}\end{aligned}$$

(6) 和, 差を積になおす公式

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

微分法の公式

関数	導関数	関数	導関数
$af(x) + bg(x)$ (a, b は定数)	$af'(x) + bg'(x)$	$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$	$f(g(x))$	$f'(g(x))g'(x)$

基本的な関数の導関数

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
x^α	$\alpha x^{\alpha-1}$	e^x	e^x
a^x	$a^x \log a$	$\log x $	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x \log a}$	$\sin x$	$\cos x$
$\cos x$	$-\sin x$	$\tan x$	$\frac{1}{\cos^2 x}$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$		

積分法の公式

(1) 置換積分 $x = g(t)$ のとき

$$\int f(x) dx = \int f(g(t))g'(t) dt$$

(2) 部分積分

$$\int f'(x)g(x) dx = f(x)g(x) - \int f(x)g'(x) dx$$

(3) その他

$$\int \frac{f'(x)}{f(x)} dx = \log |f(x)|$$

基本的な関数の不定積分 (積分定数は省略)

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
$x^\alpha \ (\alpha \neq -1)$	$\frac{x^{\alpha+1}}{\alpha+1}$	$\sin^{-1} x$	$x \sin^{-1} x + \sqrt{1-x^2}$
$\frac{1}{x}$	$\log x $	$\cos^{-1} x$	$x \cos^{-1} x - \sqrt{1-x^2}$
e^x	e^x	$\tan^{-1} x$	$x \tan^{-1} x - \frac{1}{2} \log(1+x^2)$
$\sin x$	$-\cos x$	$\frac{1}{x^2+a^2} \ (a > 0)$	$\frac{1}{a} \tan^{-1} \frac{x}{a}$
$\cos x$	$\sin x$	$\frac{1}{\sqrt{x^2+a}} \ (a \neq 0)$	$\log x + \sqrt{x^2+a} $
$\tan x$	$-\log \cos x $	$\sqrt{x^2+a}$	$\frac{1}{2} (x\sqrt{x^2+a} + a \log x + \sqrt{x^2+a})$
$\log x $	$x(\log x - 1)$	$\frac{1}{x^2-a^2} \ (a > 0)$	$\frac{1}{2a} \log \left \frac{x-a}{x+a} \right $
$a^x \ (a > 0, a \neq 1)$	$\frac{a^x}{\log a}$	$\frac{1}{\sqrt{a^2-x^2}} \ (a > 0)$	$\sin^{-1} \frac{x}{a}$
		$\sqrt{a^2-x^2} \ (a > 0)$	$\frac{1}{2} (x\sqrt{a^2-x^2} + a^2 \sin^{-1} \frac{x}{a})$

いくつかの無理数の近似値

- (1) $\pi = 3.14159\dots$ (2) $e = 2.71828\dots$ (3) $\sqrt{2} = 1.41421\dots$ (4) $\sqrt{3} = 1.73205\dots$
 (5) $\sqrt{5} = 2.23607\dots$ (6) $\log 2 = 0.69314\dots$ (7) $\log 3 = 1.09861\dots$ (8) $e^{-1} = 0.36787\dots$