Calculation for Eigen Value of Matrix Given matrix; A

(% i1) A:matrix([-13,-8,-4],[12,7,4],[24,16,7]);

$$\begin{pmatrix} -13 & -8 & -4 \\ 12 & 7 & 4 \\ 24 & 16 & 7 \end{pmatrix} \tag{A}$$

Normarizing matrix(non-singaluar); P

(% i2) P:matrix([1,1,2],[-2,-1,-3],[1,-2,0]);

$$\begin{pmatrix} 1 & 1 & 2 \\ -2 & -1 & -3 \\ 1 & -2 & 0 \end{pmatrix} \tag{P}$$

Alternate matrix of B for the same result;

(% i3) B:matrix([-1,0,0],[0,3,0],[0,0,-1]);

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{B}$$

Inverse matrix;

(% i4) PV:invert(P);

$$\begin{pmatrix} -6 & -4 & -1 \\ -3 & -2 & -1 \\ 5 & 3 & 1 \end{pmatrix} \tag{PV}$$

Normalizing P^(-1).A.P;

(% i5) PV.A.P;

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{\% o5}$$

Characteristic Polynomial of A using varible x;

(% i6) charpoly(A,x);

$$\left((7-x)^2-64\right)(-x-13)+8\,(12\,(7-x)-96)-4\,(192-24\,(7-x)) \tag{\% o6}$$

(% i7) expand(charpoly(A,x));

$$-x^3+x^2+5x+3 \tag{\% o7}$$

(% i8) PHA:determinant(A - matrix([x,0,0],[0,x,0],[0,0,x]));

$$\left((7-x)^2-64\right)(-x-13)+8\,(12\,(7-x)-96)-4\,(192-24\,(7-x)) \quad \text{(PHA)}$$

we get the characteristic polinomial of A;

(% i9) expand(PHA);

$$-x^3+x^2+5x+3 \quad \text{(\% o9)}$$

To solve the eigen value;

(% i10) factor(expand(PHA));

$$-(x-3)\,(x+1)^2 \quad \text{(\% o10)}$$

Here we get three x=3 (single) and x=-1 (double) for matrix A;

(% i11) eigenvalues(A);

$$[[3,-1],[1,2]] \quad \text{(\% o11)}$$

The solution of B, which is same as A;

(% i12) eigenvalues(B);

$$[[-1,3],[2,1]] \quad \text{(\% o12)}$$

(% i13) eigenvectors(A);

$$[[[3,-1],[1,2]],[[[1,-1,-2]],[[1,0,-3],[0,1,-2]]]] \quad \text{(\% o13)}$$

(% i14) eigenvectors(B);

$$[[[-1,3],[2,1]],[[[1,0,0],[0,0,1]],[[0,1,0]]]] \quad \text{(\% o14)}$$

Unit matrix of degree 3;

(% i15) Ei3:matrix([1,0,0],[0,1,0],[0,0,1]);

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} \quad \text{(Ei3)}$$

Hamilton's theorem; the result equals the zero matrix;

(% i16) A^^3 - A^^2 - 5*A-3*Ei3;

$$\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix} \quad \text{(\% o16)}$$