Motives in Tokyo on the occasion of Shuji Saito's 60th birthday

Date: March 26 (Mon)–March 30 (Fri) 2018

Place: Lecture Hall, Graduate school of Mathematical Sciences, The University of Tokyo

Organizers: M. Asakura, T. Geisser, N. Otsubo, K. Sato, T. Terasoma

March 26th (Monday)

10:00–11:00 K. Kato, Heights of motives and extended period domains

11:15–12:15 T. Saito, Characteristic cycle of an ℓ-adic sheaf

14:00–15:00 G. Tamme, Excision for localizing invariants

15:30–16:30 M. Kerz, Negative algebraic K-theory

16:45–17:45 B. Kahn, Albanese kernels and Griffiths groups

March 27th (Tuesday)

10:00–11:00 L. Hesselholt, K-theory and L-theory

11:15–12:15 C. Haesemeyer, On the K-theory of line bundles

14:00–15:00 K. Fujiwara, Cohomological purity from perfectoid viewpoint

15:30–16:30 O. Wittenberg, On the Lüroth problem for real varieties

16:45–17:45 M. Morrow, Motivic filtrations on K-theory and topological Hochschild homology

18:15 banquet

March 28th (Wednesday)

10:00–11:00 S. Kelly, A more streamlined comparison of cdh and ldh cohomology

11:15–12:15 A. Krishna, Zero cycles and K-theory on schemes

14:00–15:00 J.-L. Colliot-Thélène, Disproving stable rationality (survey)

15:30–16:30 M. Levine, Quadratic Euler characteristics

16:45–17:45 J. Ayoub, Proof of the conservativity conjecture

March 29th (Thursday)

10:00–11:00 T. Yamazaki, Mixed Hodge structures with modulus

11:15–12:15 F. Binda, Rigidity for relative 0-cycles

14:00–15:00 K. Rülling, Reciprocity sheaves and conductors

15:30–16:30 J. Fresán, Hodge theory of Kloosterman connections

16:45–17:45 P. A. Østvær, Bott inverted algebraic cobordism

March 30th (Friday)

10:00–11:00 T. Abe, Ramification theory and homotopies

11:15–12:15 H. Esnault, Rigid local systems and integrality

14:00–15:00 S. Bloch, Motivic Gamma functions

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Abstracts

T. Abe:

I will report on the on-going project with D. Patel on characteristic cycles and epsilon factors. I will first recall de Rham epsilon factor after Beilinson and Patel, and talk about the localization formula. A goal of our project is to realize this picture in the ℓ-adic setting. I will talk about possible strategy, and formulate a conjecture which might be useful to construct a theory of epsilon factors.

J. Ayoub:

I'll give an overview of the proof of the conservativity conjecture for the classical realisations of mixed motives in characteristic zero.

F. Binda:

In this talk, we will present a relation between the classical Chow group of 1-cycles on a regular scheme $\mathcal{X}$, projective and flat over an excellent Henselian discrete valuation ring A with perfect residue field k, and the so-called cohomological Chow group of zero cycles of the special fiber, introduced by Levine and Weibel in the 1980s. If k is algebraically closed and with finite coefficients (prime to the residue characteristic) these groups turn out to be isomorphic. This generalizes a previous argument due to Bloch and Esnault-Kerz-Wittenberg to the case of regular models with arbitrary reduction. From this, one can re-prove in case of bad reduction that the etale cycle class map for relative 0-cycles with finite coefficients on $\mathcal{X}$ is an isomorphism, a result due to Saito-Sato in the case of (quasi) semi-stable reduction. If time permits, we will discuss how the techniques can be extended to the case where $\mathcal{X}$ is only quasi-projective, obtained from a projective scheme $\overline{\mathcal{X}}$ by removing a horizontal SNC divisor, giving what we might call a proper base change theorem "with compact support" for relative 0-cycles. This is a joint work with Amalendu Krishna.

S. Bloch:

The study of solutions of Picard-Fuchs differential equations in the neighborhood of singularities with maximal unipotent monodromy (MUM) leads to Mellin transforms which resemble the classical Gamma function. This is joint work with M. Vlasenko based on ideas of V. Golyshev and D. Zagier.

J.-L. Colliot-Thélène:

For more and more classes of rationally geometricallly connected varieties, specialisation methods have established nonstable rationality for very general members of the class. In recent work of Schreieder, higher unramified cohomology and Pfister quadrics of arbitrary dimension play a key role. This leads to striking results on Fano hypersurfaces.

H. Esnault:

(joint work with Michael Groechenig) We prove that a cohomologically rigid complex local system with finite determinant and quasi-unipotent monodromies at infinity is integral. This answers positively a conjecture by Carlos Simpson. More generally Simpson conjectures that rigid complex local systems are of geometric origin. We'll show how to use Langlands' correspondence to treat (part of) Simspon's conjectures.

J. Fresán:

Recently, Broadhurst studied the L-functions associated with symmetric powers of Kloosterman sums and conjectured a functional equation after extensive numerical experiments. I will explain how the irregular Hodge filtration allows one to explain the gamma factors at infinity in a similar way to Serre's recipe for usual motives. Based on work in progress with Claude Sabbah and Jeng-Daw Yu.

K. Fujiwara:

For a given cohomology theory, the cohomological purity, i.e., the vanishing of local cohomology groups in low codimension, is a basic question. In this talk I plan to discuss purity of étale and flat cohomologies of regular schemes in the mixed characteristic case via perfectoid methods.

C. Haesemeyer:

We discuss joint work with C. Weibel analysing the *K*-theory of (the total space a geometric) line bundle relative to the base, viewed as a generalisation of Bass' Nil *K*-groups. In particular we will talk about one of the core technical ingredients, a globalisation of the Kassel-Sletsjœspectral sequence for Hochschild homology.

L. Hesselholt:

Arrows have a very particular property: They can be reversed. More precisely, the infinity-category of infinity-categories has one non-trivial automorphisms, which takes an infinity-category to its opposite infinity category. In this talk, I will explain how this structure interacts with *K*-theory.

B. Kahn:

We describe the Griffiths group of the product of a curve C and a surface S as a quotient of the Albanese kernel of S over the function field of C. When C is a hyperplane section of S varying in a Lefschetz pencil, we prove the nonvanishing in Griff$(C \times S)$ of a modification of the graph of the embedding $C \hookrightarrow S$ for infinitely many members of the pencil, provided the ground field k is of characteristic 0, the geometric genus of S is > 0, and k is large or S is "of motivated abelian type".

K. Kato:

I explain ideas on heights of motives. I explain how we can formulate the motive versions of Vojta conjectures and Manin conjectures for motives, looking at analytic theories of period domains and toroidal partial compactifications of period domains.

S. Kelly:

Half of the proof of the main result of Ast. 391 is a comparison of the cdh and ldh cohomologies of certain "nice" presheaves. There, "nice" is an awkward list of very special properties that don't give much insight into why the cohomologies should agree. Worse, the proof is a poorly structured pile of lemmas, which are difficult to arrange into some kind global narrative. In this talk we present a clearer proof that the cdh and ldh cohomologies of the relevant presheaves agree, isolating universal homeomorphism invariance of the presheaves as the key ingredient (in addition to a structure of traces). The study of finite flat algebras over (not necessarily Nagata) henselian valuation rings plays an important role.

M. Kerz:

In the talk I will explain a proof of Weibel's conjecture on the vanishing of negative algebraic *K*-groups (joint work with F. Strunk and G. Tamme).

A. Krishna:

The relation between 0-cycles and *K*-theory (especially, the Grothendieck group of vector bundles) is a well established theory for smooth varieties. In recent times, a theory of cycles with modulus has emerged, and these cycles are expected to be directly related to the relative *K*-theory. In this talk, we shall present some results in this direction. In particular, we shall see that the picture for 0-cycles with modulus is now sufficiently complete for affine schemes. An important tool in this study is the theory of 0-cycles on singular schemes. We shall show how these are all related.

M. Levine:

We will describe a series of interpretations of the "motivic" Euler characteristic of a smooth projective variety over a perfect field k. The motivic Euler characteristic an element of the Grothendieck-Witt ring of quadratic forms over k; we will show how this refines the usual topological Euler characteristic and how one can compute it using Euler classes in various cohomology theories.

M. Morrow:

In joint work with Bhargav Bhatt and Peter Scholze we constructed a motivic filtration on the topological Hochschild and cyclic homologies of a class of characteristic p (or, more generally, p-adic) rings; the graded pieces, i.e., the corresponding motivic cohomologies, are given by a form of derived crystalline or syntomic cohomology. Combining this with joint work with Dustin Clausen and Akhil Mathew, in which we identify p-adic étale *K*-theory with topological cyclic homology, leads to an Atiyah-Hirzebruch-style filtration on p-adic *K*-theory in new settings. In particular, in the context of

a smooth variety X with modulus D, this produces a motivic filtration on the wild part of the relative K-theory. I will give an overview of these results and of the techniques used.

P. A. Østvær:
In joint work with Elden Elmanto, Marc Levine, and Markus Spitzweck we compare the stable motivic homotopy category SH with its etale analogue $\mathrm{SH}_{\mathrm{et}}$. By inverting a Bott element in algebraic cobordism we obtain an equivalence with the corresponding etale theory. Our method of proof amplifies the effects of the Beilinson-Lichtenbaum conjecture via the slice spectral sequence.

K. Rülling:
(joint work with Shuji Saito) For a general reciprocity Nisnevich sheaf as defined by Kahn-Saito-Yamazaki we define a motivic conductor. We give a list of properties which uniquely determine it and use this to show that in many examples this conductor coincides with classical abelian conductors, such as the Artin conductor of a rank 1 representation, the conductor on an algebraic group defined by the Rosenlicht-Serre local symbol or the irregularity of a rank one connection. We also give motivic versions of some of the more involved properties of the classical conductors as studied by Kato and others.

T. Saito:
The characteristic cycle of an ℓ-adic sheaf on a smooth variety over a perfect field is a $\mathbf{Z}$-linear combination of irreducible components of the singular support, defined by Beilinson as a closed conical subset of the cotangent bundle. We discuss its functorial property with respect to proper direct image including the index formula.

G. Tamme:
A fundamental property of algebraic K-theory is that it is a localizing invariant, i.e., it sends a Verdier quotient sequence $A \to B \to B/A$ of small dg categories or small stable infinity-categories to a fibre sequence of spectra. In the talk I will explain that any localizing invariant also sends certain pullback diagrams of categories to pullback squares of spectra. If one applies this to categories of perfect complexes over rings, one obtains a general descent result which includes Nisnevich descent and Suslin's excision for algebraic K-theory as special cases.

O. Wittenberg:
(joint work with Olivier Benoist.) The Lüroth problem asks whether unirational varieties are rational. It has a positive answer for complex curves and surfaces; negative answers for complex threefolds have been understood since the 70's. I will discuss the Lüroth problem for real algebraic varieties that are geometrically rational and explain a counterexample not accounted for by the topology of the real locus or by a nontrivial unramified cohomology group over the reals.

T. Yamazaki:

We define a notion of mixed Hodge structure with modulus that generalizes the classical notion of mixed Hodge structure introduced by Deligne and the level one Hodge structures with additive parts introduced by Kato and Russell in their description of Albanese varieties with modulus. With modulus triples of any dimension we attach mixed Hodge structures with modulus. We combine this construction with an equivalence between the category of level one mixed Hodge structures with modulus and the category of Laumon 1-motives to generalize Kato-Russell's Albanese varieties with modulus to 1-motives.