

BOUNDARY VALUE PROBLEMS WITH FRACTIONAL POWER SINGULARITIES

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Fractional coordinate transformations are useful to analyze microlocally some boundary value problems, whose characteristic roots degenerate just on the boundary $\{x_1 = 0\}$. More precisely, let P be an m -th order operator with real analytic coefficient, whose principal symbol $\sigma(P)(x, \zeta)$ has the following form in a neighborhood of $(0; 0, i\eta')$:

$$\sigma(P)(x, \zeta) = \prod_{j=1}^m (\zeta_1 - x_1^\nu \alpha_j(x, \zeta')).$$

Here ν is a positive integer. Then, the singular coordinate transformation

$$y_1 = \left(\frac{x_1}{\nu + 1} \right)^{1/(\nu+1)}, \quad y_j = x_j \quad (j = 2, \dots, n)$$

is useful to blow up the multiplicities of characteristic roots on $\{x_1 = 0\} = \{y_1 = 0\}$. Indeed, by using such transformations, Professor Y. Chiba succeeded in explicit constructions of microlocal solutions with pure singularities in the case that $\alpha_j(x, \zeta')$'s are pure-imaginary-valued and distinct from each other. We justify these fractional calculations in general microlocal boundary value problems for hyperfunctions; in particular, we can consider operators of the following type:

$$P(x_1^{1/(\nu+1)}, x', x_1 \partial_{x_1}, \partial_{x'}).$$

Further we will give some applications to mixed boundary value problems which degenerate on the boundary.