Complementary and Degradable Channels in Quantum Information Theory

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Abstract: In qauntum information theory, noise is represented by a completely positive trace preseving map, typically referred to as a "channel". Using Stinespring's reprentation theory and Arveson's commutant lifting theorem, the complement Φ^C of a channel Φ can be defined and shown to represent the environment's view. A channel N is called degradable if there is another channel X whose action following that of the channel yields the complement, i.e., there is a channel X such that $X \circ N = N^C$.

Both degradable and complementary channels have implications for the following important question.
When can the (asympotitic) capacity be reduced to a a "one-short" formula, as in classical information theory ? It should be noted that quantum Shannon theory is much richer with many different types of capacity, some of which can not always be reduced to a simple formula. This talk will try to give a flavor for this subject and an indication of the many challenging mathematical questions remaining.