Title: Concave functions and symmetric norms

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Abstract:

Given $A_i \geq 0$ and $Z_i \geq 0$ for all $i = 1, \dots, n$, we have this theorem:

$$|| \sum f(Z_i A_i Z_i)|| \leq || \sum Z_i f(A_i) Z_i||.$$

This theorem contains the next two well-known recent inequalities: Let A, B, Z be positive semidefinite matrices of same size and suppose Z is expansive, i.e., $Z \geq I$. Two remarkable inequalities are

$$\|f(A+B)\| \leq \|f(A)+f(B)\| \quad \text{and} \quad \|f(ZAZ)\| \leq \|Zf(A)Z\|$$

for all non-negative concave function f on $[0,\infty)$ and all symmetric norms $\|\cdot\|$ (in particular for all Schatten p-norms). In this paper we survey several related results and we show that these inequalities are two aspects of a unique theorem. For the operator norm, our result also holds for operators on an infinite dimensional Hilbert space.