Self-extensions and prime factorizations for simple $U_q(L\mathfrak{sl}_2)$-modules

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Abstract

V: finite-dimensional simple $U_q(L\mathfrak{g})$-module

$V \cong V_1 \otimes \cdots \otimes V_r$: prime factorization

$\Rightarrow$ **?** $\dim \mathrm{Ext}^1(V, V) = r$

OK for $U_q(L\mathfrak{sl}_2)$ with the assumption $V_i \not\cong V_j$ $(i \neq j)$

Prime factorization

$U_q(L\mathfrak{g})$: quantum loop algebra ($q \neq$ root of unity)

Def

V: nontrivial f. d. simple $U_q(L\mathfrak{g})$-module is **prime** if

$V \cong V_1 \otimes V_2 \Rightarrow V_1$ or V_2 is trivial

$\rightsquigarrow$ **prime factorization**

Known results

$\mathfrak{g} = \mathfrak{sl}_2$

- $\{$prime modules$\}$ = $\{$evaluation modules$\}$
- $\exists$ combinatorial description of prime factorization

($\to$ unique in this case)

General $\mathfrak{g}$

- $\{$prime modules$\} \supsetneq \{$minimal affinizations$\}$
  $\supsetneq \{$Kirillov-Reshetikhin modules$\}$

Recent conjecture

Conj [Chari-Moura-Young arXiv:1112.6376]

V is prime $\Leftrightarrow \dim \mathrm{Ext}^1(V, V) = 1$

Thm [CMY]

OK for $\mathfrak{g} = \mathfrak{sl}_2$

Naive refinement

$V \cong V_1 \otimes \cdots \otimes V_r$: prime factorization

$\Rightarrow$ **?** $\dim \mathrm{Ext}^1(V, V) = r$

Evidence

[CMY] $\mathfrak{g} = \mathfrak{sl}_2$

- $\dim \operatorname{Ext}^1(V, V) = 1$ for V: evaluation module

- $V \cong V_1 \otimes \cdots \otimes V_r$: prime factorization

  - $\operatorname{Ext}^1(V_i, V_i) \hookrightarrow \operatorname{Ext}^1(V, V)$

  - $V_i \not\cong V_j \ (i \neq j) \Rightarrow \bigoplus_{i=1}^{r} \operatorname{Ext}^1(V_i, V_i) \hookrightarrow \operatorname{Ext}^1(V, V)$

$\therefore \dim \operatorname{Ext}^1(V, V) \geq r$ if $V_i \not\cong V_j \ (i \neq j)$

Upper bound

[K] $\mathfrak{g} = \mathfrak{sl}_2$

$V \cong V_1 \otimes \cdots \otimes V_r$: prime factorization

$\Rightarrow$ $\boxed{\dim \mathrm{Ext}^1(V, V) \leq r}$

$\therefore \dim \mathrm{Ext}^1(V, V) = r$ if $V_i \not\cong V_j$ $(i \neq j)$

Proof for $r = 1$

V: evaluation module $\Rightarrow \dim \mathrm{Ext}^1(V, V) \leq 1$

$$\mathrm{Ext}^1(V, V) \cong \mathrm{Ext}^1(\mathrm{triv}, V \otimes V^*)$$

$$0 \to \mathrm{triv} \to V \otimes V^* \to M \to 0$$

$$\downarrow \mathrm{Hom}(\mathrm{triv}, -)$$

$$0 = \mathrm{Ext}^1(\mathrm{triv}, \mathrm{triv}) \to \mathrm{Ext}^1(\mathrm{triv}, V \otimes V^*) \to \underbrace{\mathrm{Ext}^1(\mathrm{triv}, M)}_{\textbf{1-dim}}$$

Remark

$\mathfrak{g} = \mathfrak{sl}_2$

V: evaluation module $\Rightarrow$ $V^{\otimes 2}$: simple

[CMY] $\dim \mathrm{Ext}^1(V^{\otimes 2}, V^{\otimes 2}) \geq 2$

(In particular $\dim \mathrm{Ext}^1(V^{\otimes 2}, V^{\otimes 2}) = 2$)

If $\dim \mathrm{Ext}^1(V^{\otimes r}, V^{\otimes r}) \geq r$ holds,

then the naive conjecture is true for $\mathfrak{g} = \mathfrak{sl}_2$