Introduction to deformation quantization

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   - 2.1 Poisson algebra
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   - 3.1 Deformation quantization
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