Introduction to deformation quantization

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 - 1.2 Proposal of describing geometry in terms of noncommutative rings
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 - 2.2 Examples of Poisson algebras
- 3. Deformation quantization
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 - 3.2 Moyal product
 - 3.3 Remarks on deformation quantization
- 4. Hochschild cohomology
 - 4.1 Fundamental properties of hochschild cohomologies
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- 5. Equivalence of deformation quantizations
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 - 5.2 Auto-equivalence
- 6. Deligne's relative classes
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 - 6.3 Surjectivity of {Deformation quantizations} \rightarrow {Deligne's relative classes}
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 - 10.2 L_{∞} -algebras
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 - 10.4 Compactifications and the existence of the L_{∞} -map in the case of $M = \mathbb{R}^n$
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- 11. Path integral expression of the deformation quantization formula
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 - 11.1.3 Treatment of the Poisson-sigma model as a gauge theory and the necessity of the BV-formalism
 - $11.2\,$ The Batalin-Vilkovisky (BV) formalism
 - 11.2.1 Determination of the space of fields,
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- 11.2.4 Quantum BV master equation,
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- 11.3 Quantization of the Poisson-sigma model in terms of the BV-formalism
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 - 13.2 Convergence of the deformtation quantization of Fréchet Poisson algebras
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