解答

第0章

問 1.1 $\sin\dfrac{\pi}{8}=\dfrac{\sqrt{2-\sqrt{2}}}{2}$, $\cos\dfrac{\pi}{8}=\dfrac{\sqrt{2+\sqrt{2}}}{2}$, $\tan\dfrac{\pi}{8}=\sqrt{2}-1$,

問 1.2 省略. **問 2.1** 省略.

問 2.2 (1) $\dfrac{1}{3}$ (2) 2 (3) $\dfrac{1}{16}$ (4) $\dfrac{1}{16}$

問 2.3 (1) $\sqrt[3]{2}, \sqrt[4]{3}, \sqrt[5]{4}$ (2) $5^{-1}, 5^{\frac{-2}{3}}, 5^{0}, 5^{\frac{1}{3}}, 5^{\frac{1}{2}}$ (3) $\sqrt[4]{0.125}, \sqrt[3]{0.25}, \sqrt{0.5}$

問 2.4 1.

問 2.5 $x_0=3, x_1=2, x_2=\dfrac{7}{4}, x_3=\dfrac{97}{56}, x_4=\dfrac{18817}{10864}=1.7320508\cdots$

問 3.1 省略.

問 3.2 (1) (c) (2) (b) (3) (a) (4) (d).

問 3.3 (1) $a=2, c=2$ (2) $a=\dfrac{1}{3}, c=-1$

問 4.1 省略. **問 4.2** 省略. **問 5.1** 省略.

問 5.2 $a=2, b=-6, d=3$.

問 5.3 省略.

問 6.1 $m=\sqrt{2}$.

問 6.2 省略. **問 6.3** 省略. **問 7.1** 省略.

問 7.2 $\left(\dfrac{3}{4}, \pm\dfrac{3\sqrt{3}}{4}\right)$

第0章 練習問題

1.

(1) $(x+1)^2$ (2) $2(x+2)^2-7$ (3) $2\left(x+\dfrac{3}{2}\right)^2-\dfrac{7}{2}$

(4) $-3\left(x-\dfrac{1}{3}\right)^2-\dfrac{2}{3}$ (5) $-\left(x+\dfrac{3}{2}\right)^2+\dfrac{21}{4}$ (6) $\left(x-\dfrac{3}{2}\right)^2-\dfrac{5}{4}$

(7) $\left(x-\dfrac{3}{2}\right)^2-\dfrac{1}{4}$ (8) $-2\left(x+\dfrac{1}{4}\right)^2+\dfrac{9}{8}$

2. (1) $y = x - 2$ (2) $y = x^2 + 1$ (3) $y = \cos\left(x - \frac{\pi}{4}\right) + 1$
(4) $y = 2\sqrt{x-2} + 1$ (5) $y = 2^{x+1} + 2$ (6) $y = \frac{-1}{4x}$

3. (1) $\frac{\pi}{6}$ (2) $\frac{7}{18}\pi$ (3) $\frac{16}{9}\pi$ (4) $\frac{35}{9}\pi$ (5) 8π (6) $\frac{-\pi}{4}$ (7) $\frac{-\pi}{12}$ (8) 0

4. (1) $\frac{\sqrt{3}}{2}$ (2) $\frac{-\sqrt{3}}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) 1 (5) $\frac{\sqrt{3}}{2}$ (6) $\frac{1}{\sqrt{2}}$
(7) $\frac{-1}{2}$ (8) $\frac{-1}{\sqrt{3}}$ (9) $\frac{1}{\sqrt{2}}$ (10) $\frac{-1}{\sqrt{2}}$ (11) $-\sqrt{3}$ (12) 1

5. (1) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (2) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (3) $-2-\sqrt{3}$ (4) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (5) $2-\sqrt{3}$ (6) $\frac{1+\sqrt{3}}{2\sqrt{2}}$
(7) $\frac{2-\sqrt{2}}{4\sqrt{2}}$ (8) $\frac{1+\sqrt{3}}{4\sqrt{2}}$

6. (1) 125 (2) 2 (3) -0.5 (4) 0.1 (5) 1 (6) $\frac{125}{2}$
(7) $\frac{1}{9}$ (8) $a^{\frac{5}{6}}b^{\frac{1}{6}}$

7. (1) $\sqrt{\frac{1}{3}}, \sqrt[3]{9}, \sqrt[4]{27}, 3$ (2) $\sqrt[4]{35}, \sqrt{6}, \sqrt[3]{15}$ (3) $\sqrt[3]{2}, \sqrt[6]{5}, \sqrt[4]{3}$
(4) $\frac{1}{3}, \frac{1}{\sqrt{3}}, \sqrt[3]{9}, \sqrt{27}$ (5) $\sqrt[10]{10}, \sqrt[3]{2}$ (6) $\sqrt[4]{7}, \sqrt{3}, \sqrt[3]{6}$
(7) $\sqrt[6]{5}, \sqrt[4]{3}$

8. (1) 3 (2) $\frac{3}{4}$ (3) -1 (4) $\frac{1}{2}$ (5) 5 (6) $\frac{3}{4}$
(7) $\sqrt{3}$ (8) 125

9. (1) $1-a$ (2) $\frac{2a}{b}$ (3) $\frac{a}{b}$ (4) $\frac{4a+2b}{b}$
(5) $\frac{2a}{1-a}$ (6) $\frac{6a}{1-a}$ (7) $\frac{b-1}{2}$ (8) $\frac{2-2a}{2a+b}$

10. (1) $\frac{1}{8}$ (2) 2 (3) $\log_2 10$ (4) $\frac{5}{4}$ (5) 2, $\frac{1}{4}$ (6) 1

第 1 章

問 1.1 (1) 0 (2) 1 (3) 0 (4) 収束しない

問 1.2 (1) e^6 (2) $\frac{1}{e}$

問 2.1 省略. **問 2.2** 省略. **問 2.3** 収束する. **問 2.4** 収束する.

問 3.1 (1) 定義域 $(-\infty, \infty)$, 値域 $[0, \infty)$
(2) 定義域 $(0, \infty)$, 値域 $(-\infty, \infty)$
(3) 定義域 $(-\infty, -1) \cup (1, \infty)$, 値域 $(0, \infty)$
(4) 定義域 $\{x : x \neq (2n+1)\pi/2,\ n$ は整数 $\}$, 値域 $(-\infty, \infty)$

問 4.1 (1) $-\dfrac{1}{2}$ (2) $\dfrac{a}{b}$ (3) 1 (4) -1 (5) 8 (6) e^{km}

問 4.2 省略.

問 5.1 連続となる.

問 5.2 省略.

問 5.3 例えば, $y = \tan\pi\left(x - \dfrac{1}{2}\right)$.

問 6.1 $f(x) = \begin{cases} 0 & (0 \leqq x < 1 \text{ のとき}) \\ 1 & (x = 1 \text{ のとき}) \end{cases}$.

問 6.2 (1) 1 (2) $\dfrac{1}{2}$ (3) 4.

問 7.1 (1) $\dfrac{5}{6}\pi$ (2) $-\dfrac{\pi}{6}$

問 7.2 $\dfrac{\pi}{2}$, $-\dfrac{\pi}{2}$

問 7.3 (1) $\dfrac{\pi}{2}$ (2) $\dfrac{\pi}{4}$

問 7.4 省略.

第 1 章　練習問題

1. (1) 0 (2) $\dfrac{-3}{2}$ (3) 0 (4) 0 (5) 0 (6) ∞ (7) $\dfrac{1}{2}$ (8) $\dfrac{1}{2}$
(9) 2 (10) 0 (11) 2 (12) e (13) 1 (14) 0 (15) 0

2. (1) 発散 (2) 収束 (3) 収束 (4) 発散 (5) 発散 (6) 収束
(7) 収束 (8) 収束 (9) 収束 (10) 発散 (11) 発散 (12) 収束

3. (1) 0 (2) 7 (3) $\dfrac{1}{2}$ (4) e^2 (5) e^6 (6) e^2 (7) $\dfrac{1}{4}$
(8) 1 (9) $\dfrac{5}{3}$ (10) $\dfrac{1}{2}$ (11) 0 (12) 1 (13) 3 (14) -1
(15) $a > 1$ のとき 1, $a = 1$ のとき 0, $0 < a < 1$ のとき -1

4, 5, 6. 省略

7. (1) 1 (2) ∞ (3) $\dfrac{1}{2}$ (4) $\dfrac{1}{e}$ (5) $\dfrac{1}{e}$ (6) 8

8. $y = 2 + 2x - 2\sqrt{2 + 2x}$.

9. (1) $\dfrac{\pi}{3}$ (2) $\dfrac{-\pi}{4}$ (3) $\dfrac{\pi}{6}$ (4) $\dfrac{\sqrt{3}}{2}$ (5) 1
(6) $\dfrac{1}{2}$ (7) $\dfrac{4\sqrt{6}}{25}$ (8) $\dfrac{1}{3}$ (9) 0 (10) $\dfrac{3}{4}\pi$
(11) $\dfrac{\pi}{2}$ (12) $\dfrac{-56}{33}$ (13) $\dfrac{-\pi}{2}$ (14) $\dfrac{2}{3}\pi$ (15) $\dfrac{\pi}{3}$

10. 省略

第 2 章

問 1.1 $f'_+(0) = 1$, $f'_-(0) = -1$

問 2.1　(1) $\dfrac{-x^2-4x+1}{(x^2+1)^2}$　(2) $\dfrac{2x^3+7x^2+4x-3}{(x+2)^2}$　(3) $\dfrac{-4x^3+4}{(x^3+x+2)^2}$

問 2.2　(1) $16(x^2+1)^7x$　(2) $10x^9(x^2+1)^4(2x^2+1)+32x^7(x^8+1)^3$

問 2.3　省略.

問 2.4　(1) $-\sin x\cos(\cos x)$　(2) $2x\cos x^2$　(3) $\sin 2x$

問 3.1　$\left(\log x\cos x+\dfrac{\sin x}{x}\right)x^{\sin x}$

問 3.2　$-\dfrac{\pi}{2}+2n\pi<x<\dfrac{\pi}{2}+2n\pi$ のとき 1 ; $\dfrac{\pi}{2}+2n\pi<x<\dfrac{3}{2}\pi+2n\pi$ のとき -1 ; $x=\dfrac{\pi}{2}+n\pi$ では微分可能ではない.

問 3.3　省略.

問 4.1　(1) 3!　(2) 0　(3) 13!

問 5.1　省略.

問 6.1　(1) $\dfrac{2}{3}$　(2) $\dfrac{a}{b}$　(3) 2　(4) 2　(5) 1　(6) 1

問 7.1　(1) $x=0$ で極大値 $0, x=\dfrac{2}{3}$ で極小値 $-\dfrac{4}{27}$　(2) $x=1$ で極小値 $\dfrac{3}{2}$

(3) $x=e$ で極大値 $e^{\frac{1}{e}}$　(4) $x=\dfrac{1}{3}$ で極大値 $\left(\dfrac{1}{3}\right)^{\frac{1}{3}}\left(\dfrac{2}{3}\right)^{\frac{2}{3}}$.

問 7.2　(1) 変曲点は $(0,1)$　(2) 変曲点は $(0,0)$　(3) 変曲点は $\left(\pm\dfrac{1}{\sqrt{2}}, e^{\frac{-1}{2}}\right)$
(4) 変曲点なし

問 7.3　省略.

第 2 章 練習問題

1.

(1) $\dfrac{x}{\sqrt{x^2-1}}$　(2) $\dfrac{-x}{|x|\sqrt{1-x^2}}$　(3) $2\sqrt{1-x^2}$

(4) $\dfrac{\sqrt{x}+1}{2\sqrt{x^2+2x\sqrt{x}}}$　(5) e^{x+e^x}　(6) $\dfrac{x}{x^2+1}$

(7) $\dfrac{1}{\sqrt{1-x^2}\,(1+(\sin^{-1}x)^2)}$　(8) $\dfrac{1}{\sqrt{x^2+2}}$　(9) $\dfrac{1}{(x^2+1)\sqrt{x^2+1}}$

(10) $\dfrac{-2-x}{x^3\cos^2\frac{x+1}{x^2}}$　(11) $6x\sin^2x^2\cos x^2$　(12) $\dfrac{\cos(\log x)}{x\cos^2(\sin(\log x))}$

2.　(1) $\dfrac{\sin t}{1-\cos t}$　(2) $\dfrac{1}{\sin t}$　(3) $\dfrac{2t+\cos t}{3t^2}$　(4) $\dfrac{e^t}{2t-2}$　(5) $\dfrac{-1}{2\sin t}$　(6) $\dfrac{2\cos 2t}{e^t}$

3. (1) $(-1)^n e^{-x}$ (2) $\sin\left(x+\frac{n\pi}{2}\right)$

(3) $\frac{(-1)^{n-1}(n-1)!}{x^n}$ (4) $\frac{(-1)^{n-1}1\cdot 3\cdot 5\cdots(2n-3)}{2^n\sqrt{x^{2n-1}}}$

(5) $\frac{(-1)^n n!}{(x-1)^{n+1}}$ (6) $(x+n)^2 e^x$

(7) $\frac{(-1)^n(n-2)!}{x^{n-1}}$, $(n\geqq 2)$ (8) $x\sin\left(x+\frac{n\pi}{2}\right)+n\sin\left(x+\frac{n-1}{2}\pi\right)$

(9) $(\sqrt{2})^n e^x\cos\left(x+\frac{n\pi}{4}\right)$ (10) $\frac{(-1)^{n-4}6(n-4)!}{x^{n-3}}$, $(n\geqq 4)$

(11) $(-2)^{n-1}e^{-2x}(-2x+n)$ (12) $(-1)^n n!\left\{\frac{1}{(x-1)^{n+1}}-\frac{1}{x^{n+1}}\right\}$

4. (1) 省略. (2) n が奇数のとき，$f^{(n)}(0)=0$. n が偶数のとき，$n=2m$ とあらわすと，
$f^{(2m)}(0)=(2m-1)^2(2m-3)^2\cdots 3^2 1^2$

5. (1) $1+\frac{x^2}{2}$ (2) $81+108x+54x^2$ (3) $1+\frac{1}{2}x+\frac{3}{8}x^2$

(4) x (5) $1-2x+x^2$ (6) $1+(\log 2)x+\frac{(\log 2)^2}{2}x^2$

(7) $x+x^2$ (8) $\log 2+\frac{1}{2}x+\frac{1}{8}x^2$ (9) $1+x-x^2$

(10) $e-\frac{e}{2}x^2$ (11) 0 (12) x

6. (1) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)!}x^{2n+1}$ (2) $\sum_{n=0}^{\infty}\frac{2^n}{n!}x^n$ (3) $\sum_{n=0}^{\infty}(-1)^n x^{2n}$

(4) $\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}x^{2n+1}$ (5) $\sum_{n=0}^{\infty}\frac{(\log 3)^n}{n!}x^n$ (6) $\sum_{n=0}^{\infty}\frac{x^{n+2}}{n!}$

(7) $\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}x^n$ (8) $\sum_{n=0}^{\infty}(-1)^n x^n$ (9) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{4n}$

(10) $\sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}x^{n+2}$ (11) $\sum_{n=1}^{\infty}\frac{\sqrt{2^n}\sin\frac{n\pi}{4}}{n!}x^n$

7. (1) $\frac{1}{6}$ (2) 0 (3) 1 (4) $\log a$ (5) $\frac{-3}{2}$

(6) $\frac{7}{24}$ (7) ∞ (8) $\frac{1}{6}$ (9) $\frac{(\log 2)^2}{2}$

8. (1) $x=\frac{4}{3}$ で極小値 $-\frac{16\sqrt{3}}{9}$

(2) $x=e$ のとき極小値 e

(3) $x=\frac{1}{2}$ のとき極小値 $\frac{3}{5}$

(4) $x=2n\pi+\frac{\pi}{3}$ で極大値 $\frac{3\sqrt{3}}{4}$, $x=2n\pi-\frac{\pi}{3}$ で極小値 $-\frac{3\sqrt{3}}{4}$

(5) $x=\frac{1}{2}$ のとき極大値 $\frac{1}{2}$

(6) 極値なし

第3章

問 1.1 省略. **問 1.2** 省略.

問 2.1 (1) $\frac{1}{3}e^{x^3}$ (2) $\frac{3\cos 2x - \cos 6x}{12}$ (3) $\frac{x^2}{2} - 2\log(x^2+4)$
(4) $\log(1+e^x)$ (5) $-\log(2+\cos x)$

問 2.2 (1) $(x-1)e^x$ (2) $-x\cos x + \sin x$ (3) $x\tan^{-1}x - \frac{1}{2}\log(1+x^2)$
(4) $x\log(x^2+1) - 2(x-\tan^{-1}x)$

問 2.3 (1) $(x^2-2x+2)e^x$ (2) $\frac{e^x(\cos x + \sin x)}{2}$
(3) $-x^2\cos x + 2x\sin x + 2\cos x$

問 2.4 $-\frac{1}{4}\sin^3 x\cos x + \frac{3}{8}x - \frac{3}{16}\sin 2x$

問 2.5 (1) $I_n = -x^n e^{-x} + nI_{n-1}$ (2) $I_n = x(\log x)^n - nI_{n-1}$
(3) $I_n = \frac{a^x x^n}{\log a} - \frac{n}{\log a}I_{n-1}$

問 3.1 $\frac{1}{2}x^2 - 4x + 8\log|x+1|$.

問 3.2 (1) $2x + 7\log|x-1|$ (2) $\frac{3}{4}x^2 - \frac{13}{4}x + \frac{59}{8}\log|2x+3|$
(3) $2x + 3\tan^{-1}x$ (4) $\frac{1}{2}x^2 - 4x + \frac{5}{2}\log(x^2+4x+13) + \frac{47}{3}\tan^{-1}\left(\frac{x+2}{3}\right)$.

問 3.3 (1) $\log|x-4| + \frac{1}{2}\log|2x+3|$ (2) $\frac{1}{2}\log\left|\frac{x-1}{x+1}\right| - \tan^{-1}x$
(3) $\frac{1}{2x} + \frac{1}{2}\tan^{-1}(x-1)$.

問 3.4 (1) $\log\left|1+\tan\frac{x}{2}\right|$ (2) $\frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x$
(3) $\frac{1}{2}x + \frac{1}{4}\log\left|\frac{\tan x+1}{\tan x-1}\right|$

問 3.5 (1) $x - 2\sqrt{x} + 2\log(\sqrt{x}+1)$ (2) $\frac{2}{1+x+\sqrt{x^2+1}} + \log|x+\sqrt{x^2+1}|$
(3) $-\frac{\sqrt{1-x^2}}{x}$ (4) $\frac{a^2x}{8}\sqrt{a^2-x^2} - \frac{1}{4}x(a^2-x^2)^{\frac{3}{2}} + \frac{a^4}{8}\sin^{-1}\frac{x}{a}$

第3章 練習問題

1. (1) $\frac{1}{40}(2x+1)^{10} - \frac{1}{36}(2x+1)^9$ (2) $-\frac{1}{3}\cos(3x+1)$ (3) $\frac{1}{3}(\log x)^3$
(4) $\frac{1}{2}\log\left|\frac{e^x-1}{e^x+1}\right|$ (5) $\frac{x^2}{2}\log x - \frac{1}{4}x^2$ (6) $\frac{1}{2}(1+x^2)\tan^{-1}x - \frac{1}{2}x$
(7) $\frac{1}{4}\log\left|\frac{x-3}{x+1}\right|$
(8) $\frac{1}{4\sqrt{2}}\log\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1} + \frac{1}{2\sqrt{2}}\left\{\tan^{-1}(\sqrt{2}x+1) + \tan^{-1}(\sqrt{2}x-1)\right\}$
(9) $\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|$ (10) $\frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\frac{x}{2}$

(11) $\frac{1}{2}\log(x^2+x+4)+\frac{5}{\sqrt{15}}\tan^{-1}\left(\frac{2x+1}{\sqrt{15}}\right)$ (12) $-\cos x+\tan^{-1}\cos x$

(13) $-\frac{1}{\sin x}-\sin x$ (14) $\frac{x\{\sin(\log x)-\cos(\log x)\}}{2}$

(15) $(1+x)\tan^{-1}\sqrt{x}-\sqrt{x}$ (16) $\log\left|\frac{x}{1+\sqrt{1+x^2}}\right|$

(17) $\frac{2}{3}(1+\log x)^{\frac{3}{2}}$ (18) $\frac{1}{2}\left\{(x-2)\sqrt{4x-x^2}+4\sin^{-1}\left(\frac{x-2}{2}\right)\right\}$

2, 3. 省略.

第4章

問 1.1 (1) $\frac{\pi}{6}$ (2) $\log\left(\sqrt{2}+1\right)$

問 1.2 (1) $2xf(x^2)-f(x)$ (2) $\int_0^{x+1}f(t)\,dt+xf(x+1)$

問 1.3 (1) 1 (2) $\log\frac{5}{3}$ (3) $\frac{1}{2}\log 3-\frac{5\pi}{6\sqrt{3}}$ (4) $\frac{7}{24}$

(5) $\frac{1}{ab}\tan^{-1}\frac{a}{b}$

問 1.4 (1) $2\log 2-1$ (2) $\frac{\pi+2\log 2-2}{12}$ (3) $\frac{16}{35}$ (4) $\frac{16}{15}$ (5) $\frac{9\pi}{8}$ (6) $\frac{8}{105}$

問 1.5 省略.

問 1.6 (1) $\frac{\pi}{2}-x=t$ とせよ. (2) $a\sin x+b\cos x=\sqrt{a^2+b^2}\sin(x+\alpha)$ を使え.

問 1.7 $\frac{\pi}{4}$

問 1.8 (1) $m=n$ のとき π, $m\neq n$ のとき 0 (2) 0

問 1.9 (1) $\frac{4^n(n!)^2}{(2n+1)!}$ (2) $\frac{n!m!(b-a)^{m+n+1}}{(m+n+1)!}$

問 1.10 $0<x<1$ で $\frac{x^n}{2}<\frac{x^n}{x+1}<x^n$

問 1.11 $\frac{\pi}{4}$

問 2.1 (1) 1 (2) 存在しない (3) $\log 2$ (4) 1 (5) -1 (6) 2

(7) 6 (8) 存在しない (9) $\frac{\pi}{8}$ $\left(\text{ヒント } \frac{1}{x^4+4}=\frac{-1}{8}\frac{x-2}{x^2-2x+2}+\frac{1}{8}\frac{x+2}{x^2+2x+2}\right)$

問 2.2 (1) 30 (2) $\frac{3}{4}$ (3) $\frac{16}{315}$ (4) $\frac{4}{3}$

問 2.3 $\frac{1}{e}$

問 2.4 (1) $\frac{1}{280}$ (2) $\frac{8}{315}$ (3) $\frac{(-1)^n n!}{(m+1)^{n+1}}$

問 3.1 (1) $\frac{1}{6}$ (2) $\frac{1}{6}$

問 3.2 $\frac{16}{3}p^2$

問 3.3 $\frac{8}{15}|a|^5$

問 3.4 $\frac{64}{5}$

問 3.5 πa^2

問 3.6 $\frac{\pi a^2}{2}$

問 3.7 $2a^2$

問 3.8 (1) 1 (2) $\frac{\pi}{2}$

問 3.9 (1) $\frac{52}{3}$ (2) $\frac{4}{\sqrt{3}}$

問 3.10 (1) $2\pi^2 a$ (2) $8a$

問 3.11 $\frac{a}{2}\left\{\alpha\sqrt{\alpha^2+1}+\log(\alpha+\sqrt{\alpha^2+1})\right\}$

問 3.12 $x=\tan\theta$ とおけ.

第4章 練習問題

1. (1) $4-2\pi$ (2) $\frac{e-2}{3e}$ (3) $\frac{5\pi}{256}$ (4) $\frac{4}{3}$ (5) $\frac{1}{12}$ (6) $\frac{1+\log 2}{4}$

(7) $\frac{\pi}{9}$ (8) $\frac{\sqrt{3}-1}{2\sqrt{3}}+\frac{\pi}{36}$ (9) $\frac{\pi^2}{4}$ (10) 0 (11) $\pi\left(1-\frac{\pi}{4}\right)$ (12) $\frac{\pi}{8}\log 2$ ($x=\tan t$ とお

2. (1) $\frac{2\pi}{3\sqrt{3}}$ (2) $a\pi$ (3) $\log(\sqrt{2}+1)$ (4) $\frac{a}{a^2+b^2}$ (5) $n!$ (6) $\log 2-1$

3. (1) $\frac{x}{1+x^2}$ (2) $3e^{3x+1}$ (3) xe^x (4) $\int_0^x f(t)dt$

4. $I_n=\pi$

5. (1) 積分区間内で $1<\frac{1}{\sqrt{1-\sin x}}<\frac{1}{\sqrt{1-x}}$ が成り立つことを利用する.

(2) 積分区間内で $\sqrt{1-x^2}<\sqrt{1-x^4}<1$ が成り立つことを利用する.

(3) 自然数 k に対して, $k<x<k+1$ の範囲で $\frac{1}{(k+1)^2}<\frac{1}{x^2}$ が成り立つことを利用する.

(4) 自然数 k に対して, $k<x<k+1$ の範囲で $\frac{1}{\sqrt{x}}<\frac{1}{\sqrt{k}}$ が成り立つことを利用する.

6. (1) 24 (2) 7.161 (3) $\frac{4}{15}$ (4) π (5) $\sqrt{\pi}$ (6) $\frac{8}{15}$ (7) $\frac{1}{2}$

7. $2\tan^{-1}\frac{1}{2}$

8. $\frac{8}{3}p^2\left(1+\frac{1}{m^2}\right)^{\frac{3}{2}}$, $x=p$

9. $S=\frac{3}{8}\pi a^2$, $L=6a$

10. $\frac{8}{15}$

11. $\frac{3}{2}\pi a^2$

12. π

13. $\frac{\sqrt{5}}{2}+\frac{1}{4}\log(2+\sqrt{5})$

14. $1+\frac{\sqrt{2}}{2}\log(1+\sqrt{2})$

15. $\sqrt{2}(e^{\pi}-1)$

16. $a\left|\log\frac{y_1}{y_2}\right|$

17. (1) 2 (2) 1 (3) $e-1$ (4) $\frac{1}{2}(\log 2)^2$ (5) $2(\sqrt{2}-1)$

第5章

問 2.1 (1) $y=e^t$, (2) $y=-2e^{3t}$

問 3.1 省略

問 3.2 $38k=\log 2$ より $k=0.018240\cdots$.

問 3.3 $2e^{2k}=3$ より $k=\frac{1}{2}\log\frac{3}{2}$. よって $2e^{5\log\frac{3}{2}}=2\left(\frac{3}{2}\right)^5=15.1875g$

問 3.4 (1) $y=100e^{-t}$, (2) $y=10e^{-2t}$

問 3.5 (a) 3,850 年前, (b) 10,510 年前, (c) 7,010 年前

問 4.1 省略

問 4.2 (1) $y=e^{3t}+1$, (2) $y=e^{-2t}-1$

問 4.3 $y(1)=10+100e^{-k}=60$ より $k=\log 2$. $y(t)=10+100e^{-t\log 2}=30$ より $t=\frac{\log 5}{\log 2}=2.32194\cdots$. よって, さらに約 1.322 時間かかる.

問 4.4 $k=\frac{\log 2}{10}$. 40 分後は $y(40)=17+80\frac{1}{2^4}=22$ 度.

問 4.5 省略

問 4.6 $v=-\dfrac{mg}{k}+\left(v_0+\dfrac{mg}{k}\right)e^{-\frac{k}{m}t}$ より $v=0$ として $t=\dfrac{m}{k}\log\left(1+\dfrac{kv_0}{mg}\right)$ だから次の計算をすればよい.

$$\int_0^{\frac{m}{k}\log\left(1+\frac{kv_0}{mg}\right)}\left\{-\frac{mg}{k}+\left(v_0+\frac{mg}{k}\right)e^{-\frac{k}{m}t}\right\}dt.$$

問 4.7 (1) 省略, (2) パラシュートが開いた高度は 20.83m, 偵察機の高度は 506.61m,
(3) 運動量を計算すればよい.

問 4.8 省略

問 5.1 $x^2+y^2=1$

問 5.2 (1) $y=\dfrac{2x^2}{3x^2-1}$, (2) $y=e^{-x^2+1}$, (3) $y=\dfrac{1}{\pi}\cos^{-1}\left(-\dfrac{\pi}{3}x^3+\dfrac{\pi}{3}-1\right)$, (4) $y=\sqrt{2-x^2}$

問 6.1 e^{60} 倍

問 6.2 現在を $t=0$ とすると, $y(t)=\dfrac{100}{1+49\left(\frac{19}{49}\right)^{t+1}}$. 約 6.22 年かかる.

問 6.3 (1) $y=\dfrac{Ce^t}{Ce^t+1}$, (2) $y=C\sqrt{\dfrac{1+t}{1-t}}$

問 7.1 省略

問 7.2 省略

問 7.3 (1) $y(t)=5\sin\left(\sqrt{2}t+\dfrac{\pi}{2}\right)$, (2) $y(t)=\dfrac{\sqrt{205}}{2}\sin\left(2t+\sin^{-1}\dfrac{14}{\sqrt{205}}\right)$

第 5 章　練習問題

1. (1) $y^3=\dfrac{x^3}{Cx^3-1}$　(2) $y=\log x(y+1)+C$　(3) $\sin y=Cxe^{\frac{1}{2}x^2}$　(4) $y=Ce^{x^3}$

(5) $y=C\cos x$　(6) $y=C\left|\dfrac{\sqrt{2}x+1}{\sqrt{2}x-1}\right|^{\frac{1}{2\sqrt{2}}}$　(7) $y=\dfrac{C-x}{1+Cx}$　(8) $y=e^{Cx}$

2. (1) $y=\dfrac{1}{3}\log\left(\dfrac{3}{2}e^{2x}-\dfrac{1}{2}\right)$　(2) $y=-\log\left(1+x+\dfrac{1}{3}x^3\right)$

(3) $\cos 3y=\cos 2x$　(4) $\log|x(y-1)|=x-y-1$

3. 3536 人

4. 2400 年後 $\cdots$ 35.35%, 8000 年後 $\cdots$ 3.125%

5. 133 日

6. 省略

7. 60°C

8. 9 時 22 分

9. 午前 6 時

10. 40 m

11. 30 m $\cdots$ $\dfrac{1}{9}$, 60 m $\cdots$ $\dfrac{1}{81}$, 50 m $\cdots$ $\dfrac{1}{27}\sqrt[3]{\dfrac{1}{3}}$

12. $\sqrt{\dfrac{mg}{k}}$

13. たどりつけない

14. つける.

第6章

問 1.1 (1) $-\dfrac{6}{5}$ (2) 存在しない (3) 存在しない (4) 0

問 2.1 (1) $z_x = 6xy + 10xy^3$, $z_y = 3x^2 + 15x^2y^2$ (2) $z_x = ye^{xy}$, $z_y = xe^{xy}$

(3) $z_x = \dfrac{x}{\sqrt{x^2+y^2}}$, $z_y = \dfrac{y}{\sqrt{x^2+y^2}}$ (4) $z_x = \dfrac{|\,y\,|}{y\sqrt{y^2-x^2}}$, $z_y = -\dfrac{x}{|\,y\,|\sqrt{y^2-x^2}}$

(5) $z_x = \dfrac{1}{x\log y}$, $z_y = -\dfrac{\log x}{y(\log y)^2}$ (6) $z_x = yx^{y-1}$, $z_y = x^y \log x$

問 2.3 (1) $z_{xx} = -9\sin 3x\cos 4y$, $z_{xy} = z_{yx} = -12\cos 3x\sin 4y$, $z_{yy} = -16\sin 3x\cos 4y$

(2) $z_{xx} = \dfrac{2(y^4-x^2)}{(x^2+y^4)^2}$, $z_{xy} = z_{yx} = \dfrac{-8xy^3}{(x^2+y^4)^2}$, $z_{yy} = \dfrac{4y^2(3x^2-y^4)}{(x^2+y^4)^2}$

(3) $z_{xx} = 2(1+2x^2)e^{x^2+2y}$, $z_{xy} = z_{yx} = 4xe^{x^2+2y}$, $z_{yy} = 4e^{x^2+2y}$

(4) $z_{xx} = e^{-x}\sin y$, $z_{xy} = z_{yx} = -e^{-x}\cos y$, $z_{yy} = -e^{-x}\sin y$

(5) $z_{xx} = \dfrac{2y}{x^3}\cos\dfrac{y}{x} - \dfrac{y^2}{x^4}\sin\dfrac{y}{x}$, $z_{xy} = z_{yx} = -\dfrac{1}{x^2}\cos\dfrac{y}{x} + \dfrac{y}{x^3}\sin\dfrac{y}{x}$, $z_{yy} = -\dfrac{1}{x^2}\sin\dfrac{y}{x}$

(6) $z_{xx} = \pm\dfrac{y(2x^2-y^2)}{x^2(x^2-y^2)^{3/2}}$, $z_{xy} = z_{yx} = -\dfrac{|\,x\,|}{(x^2-y^2)^{3/2}}$, $z_{yy} = \pm\dfrac{y}{(x^2-y^2)^{3/2}}$

問 3.1 (1) $x+y-z=1$ (2) $6x+8y-z=25$ (3) $\dfrac{2}{a}x+\dfrac{2}{b}y-z=2$ (4) $2x-2y+4z=\pi$

問 4.1 (1) $-4\sin t\cos t$ (2) $e^{x-y}(1+\dfrac{1}{t^2})$ (3) $\dfrac{-2t}{\sqrt{2t^2-t^4}}$

問 4.2 (1) $z_u = 9(2u-v)$, $z_v = -9u$ (2) $z_u = 4u(u^2+v^2)$, $z_v = 4v(u^2+v^2)$

(3) $z_u = e^{xy}\dfrac{uy-vx}{u^2+v^2}$, $z_v = e^{xy}\dfrac{ux+vy}{u^2+v^2}$

問 5.2 $R_4 = \dfrac{e^{\theta y}}{4!(1+\theta x)^4}$ $(-6x^4 + 8x^3y(1+\theta x) - 6x^2y^2(1+\theta x)^2$
$+4xy^3(1+\theta x)^3 + y^4(1+\theta x)^4\log(1+\theta x))$

問 5.3 $\sqrt{2-x+y} = \sqrt{2}\left(1 - \dfrac{1}{4}(x-y) - \dfrac{1}{32}(x-y)^2 - \dfrac{\sqrt{2}(x-y)^3}{32(2-\theta(x-y))^{\frac{5}{2}}}\right)$

問 6.1 $\dfrac{dy}{dx} = \dfrac{3x^2+2x}{2y}$ $(y \neq 0)$, $\dfrac{dx}{dy} = \dfrac{2y}{3x^2+2x}$ $(x \neq 0, -\dfrac{2}{3})$, $(0,0)$ は特異点.

問 6.2 $\dfrac{dy}{dx} = -\dfrac{2x+3y}{3x+2y}$, $\dfrac{d^2y}{dx^2} = \dfrac{10}{(3x+2y)^3}$

問 6.3 $\dfrac{\partial z}{\partial x} = -\dfrac{a(x-l)}{c(z-n)}$, $\dfrac{\partial z}{\partial y} = -\dfrac{b(y-m)}{c(z-n)}$, $\dfrac{\partial^2 z}{\partial x^2} = -\dfrac{a\{c(z-n)^2 + a(x-l)^2\}}{c^2(z-n)^3}$,

$\dfrac{\partial^2 z}{\partial x\partial y} = -\dfrac{ab(x-l)(y-m)}{c^2(z-n)^3}$, $\dfrac{\partial^2 z}{\partial y^2} = -\dfrac{b\{c(z-n)^2 + b(y-m)^2\}}{c^2(z-n)^3}$

問 6.4 $\dfrac{dy}{dx} = -\dfrac{nx-lz}{ny-mz}$, $\dfrac{dz}{dx} = -\dfrac{ly-mx}{ny-mz}$

問 7.1 (1) $(x,y)=\left(\frac{1}{3},-\frac{4}{3}\right)$ で極小値 $-\frac{4}{3}$

(2) $(x,y)=(-2,0)$ で極大値 $4e^{-2}$

(3) $(x,y)=(2,2)$ で極小値 -8

(4) $(x,y)=\left(\pm\frac{1}{2},\pm\frac{1}{2}\right)$ で極小値 $-\frac{1}{8}$, $(x,y)=\left(\pm\frac{1}{2},\mp\frac{1}{2}\right)$ で極大値 $\frac{1}{8}$

問 7.3 (1) $(x,y)=\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ で極大値 $\sqrt{2}$, $(x,y)=\left(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$ で極小値 $-\sqrt{2}$

(2) $(x,y)=\left(\pm\frac{1}{\sqrt{2}},\pm\frac{1}{\sqrt{2}}\right)$ で極大値 $\frac{1}{2}$, $(x,y)=\left(\pm\frac{1}{\sqrt{2}},\mp\frac{1}{\sqrt{2}}\right)$ で極小値 $-\frac{1}{2}$

問 7.4 $(x,y)=(\pm\sqrt{2},0)$ で極大値（最大値）2, $(x,y)=(0,0)$ で極小値（最小値）0

第6章 練習問題

1. (1) 0 (2) 0 (3) 1 (4) 極限なし (5) 極限なし (6) 0

2. (1) $z_{xx}=\dfrac{4y^2(y^2-3x^2)}{(x^2+y^2)^3}$, $z_{xy}=\dfrac{8xy(x^2-y^2)}{(x^2+y^2)^3}$, $z_{yy}=\dfrac{4x^2(3y^2-x^2)}{(x^2+y^2)^3}$

(2) $z_{xx}=\dfrac{-2x^2-2xy+y^2}{(x^2+xy+y^2)^2}$, $z_{xy}=\dfrac{-x^2-4xy-y^2}{(x^2+xy+y^2)^2}$, $z_{yy}=\dfrac{x^2-2xy-2y^2}{(x^2+xy+y^2)^2}$

(3) $z_{xx}=\dfrac{(e^x+e^y)\{y^2e^{xy}-(y+1)e^{xy+x}\}-2e^x(ye^{xy}-e^{xy+x})}{(e^x+e^y)^3}$,

$z_{xy}=\dfrac{(e^x+e^y)\{e^{xy}+xye^{xy}-xe^{xy+x}\}-2e^y(ye^{xy}-e^{xy+x})}{(e^x+e^y)^3}$,

$z_{yy}=\dfrac{(e^x+e^y)\{x^2e^{xy}-(x+1)e^{xy+y}\}-2e^y(xe^{xy}-e^{xy+y})}{(e^x+e^y)^3}$

(4) $Z_{xx}=\dfrac{2xy}{(x^2+y^2)^2}$, $Z_{xy}=\dfrac{y^2-x^2}{(x^2+y^2)^2}$, $Z_{yy}=\dfrac{-2xy}{(x^2+y^2)^2}$

(5) $Z_{xx}=6x-2y$, $Z_{xy}=-2x$, $Z_{yy}=0$

3. $z_u=z(v\cos uv-\sin(u+v))$, $z_v=z(u\cos uv-\sin(u+v))$

4. 省略

5. (1) $y+xy+\dfrac{e^{\theta x}}{6}\{(x^3-3xy^2)\sin\theta y+(3x^2y-y^3)\cos\theta y\}$

(2) $1+(2x-3y)+(2x-3y)^2+\dfrac{1}{(1-2\theta x+3\theta y)^4}(2x-3y)^3$

(3) $1+\dfrac{x}{2}-\dfrac{1}{8}(x^2+4y^2)$

$+\dfrac{1}{16}\left\{x^3-6\theta x^2y^2+4xy^2(1+\theta x+2\theta^2y^2)-8(1+\theta x)\theta y^4\right\}(1+\theta x-\theta y^2)^{-\frac{5}{2}}$

6. 省略

7. $y' = -\dfrac{2x-y}{-x+2y}$

8. $y' = \dfrac{x+y}{x-y}$

9. $\dfrac{dy}{dx} = \dfrac{a-x}{y}$, $\dfrac{dz}{dx} = -\dfrac{a}{z}$

10. 省略

11. (1) $(x,y) = (2,0)$ で極小値 -4

(2) $(x,y) = \left(1, \dfrac{2}{\sqrt{3}}\right)$ で極小値 $-2\left(1 + \dfrac{8}{3\sqrt{3}}\right)$

(3) $(x,y) = (0,0)$ で極大値 0, $(x,y) = (\pm 3, \mp 3)$ で極小値 -162

(4) $(x,y) = (-1,-1)$ で極小値 9

(5) $(x,y) = (0,0)$ で極大値 1

(6) 極値なし

(7) $(x,y) = \left(\dfrac{-1}{3}, 1\right)$ で極大値 $\dfrac{32}{9}$, $(x,y) = \left(\dfrac{1}{3}, -1\right)$ で極小値 $\dfrac{-32}{9}$

(8) $(x,y) = \left(\dfrac{1}{2}, 1\right)$ で極小値 $\dfrac{5}{16}$

(9) $(x,y) = (0,0)$ で極大値 1, $(\cos\theta, \sin\theta)$, $0 \leqq \theta < 2\pi$ では極値をとるか分からない.

(10) $(x,y) = (0,0)$, $(2,4)$ で極小値はそれぞれ 0, 0

12. $(x,y) = \left(\pm\dfrac{1}{\sqrt{2}}, \pm\dfrac{1}{\sqrt{2}}\right), \left(\pm\dfrac{1}{\sqrt{2}}, \mp\dfrac{1}{\sqrt{2}}\right)$ で極小値 $\dfrac{1}{2}$, $(x,y) = (1,0), (0,1)$ で極大値 1

第 7 章

問 1.1 (1) $e^2 - 2e + 1$ (2) $\dfrac{1}{10}$ (3) $\dfrac{1}{3}$ (4) $\dfrac{1}{8}$ (5) $\dfrac{1}{6}$ (6) $\dfrac{1}{6}$

問 1.2 (1) $\dfrac{11}{84}$ (2) $\dfrac{2}{7}$ (3) $\dfrac{3}{2} - \log 2$ (4) $\dfrac{2}{3}$ (5) $\dfrac{1}{8}$ (6) $\dfrac{3\sqrt{3} + 2\pi}{18}$

問 1.4 (1) $\displaystyle\int_0^1 dy \int_{\sqrt{y}}^1 f(x,y)dx$ (2) $\displaystyle\int_0^1 dy \int_y^{\sqrt{y}} f(x,y)dx$

(3) $\displaystyle\int_0^a dy \int_{\frac{y}{2}}^y f(x,y)dx + \int_a^{2a} dy \int_{\frac{y}{2}}^a f(x,y)dy$ (4) $\displaystyle\int_a^b dy \int_y^b f(x,y)dy$

問 1.5 $\left(\dfrac{4}{3\pi}, \dfrac{4}{3\pi}\right)$

問 2.1 (1) $4(2-\sqrt{2})$ (2) $\dfrac{1}{4}$ (3) $\dfrac{\pi}{4}$ (4) $\dfrac{2}{\pi}$ (5) πa

問 3.1 (1) $\dfrac{\pi^2}{2}$ (2) $\dfrac{2}{3}\pi a^3$ (3) $\dfrac{\pi}{8}$ (4) $\dfrac{a^3}{12}$ (5) $\dfrac{2}{3}\left(\dfrac{\pi}{2} - \dfrac{2}{3}\right)a^3$

問 4.1 (1) $\dfrac{\pi}{2} - 1$ (2) $\dfrac{1}{2}\log 2 - \dfrac{5}{16}$ (3) $(e^a - 1)(e^b - 1)(e^c - 1)$

(4) $\frac{1}{4}\left(1-\frac{1}{\sqrt{3}}-\frac{\pi}{12}\right)$ (5) $\frac{1}{120}$ (6) $\frac{\pi}{12}$

問 5.1 $16a^2$

問 5.2 $\frac{1}{6}\pi(5\sqrt{5}-1)$

問 5.3 $2(\pi-2)a^2$

問 5.4 $3a^2\pi$

第 7 章　練習問題

1. (1) $(2a^2-3)\sin a+4a\cos a-a$ (2) $\frac{e-1}{2}$ (3) $\frac{e^2}{2}-e$

(4) $\frac{1}{2}\left(\frac{\pi}{4}-\frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}\right)$ (5) 1 (6) $\frac{3}{35}$

(7) $\frac{1}{2}\left(1-\frac{1}{e}\right)$ (8) $\frac{2}{9}\left(2\sqrt{2}-1\right)$ (9) 1

(10) $\frac{1}{2}\left(1-\cos\frac{\pi^2}{4}\right)$

2, 3, 4, 5. 省略

6. (1) $\frac{1}{3}$ (2) $\frac{1}{(1-\alpha)(2-\alpha)}$ (3) $\frac{1}{2}$ (4) $\frac{1}{16}\pi^2a^2$ (5) $\frac{\pi}{8}$ (6) $\frac{\pi}{16}a^4$

7. (1) $\frac{1}{2}(e^4-3e^2+2e)$ $\left(u=x+y, v=\frac{x}{y}\text{ とおく}\right)$

(2) $\frac{\pi^3}{12}$ $(u=x-y, v=x+y\text{ とおく})$

(3) $\frac{\pi}{4}\log 2$ $(x=u, y=uv\text{ とおく})$

(4) $\frac{5}{3}$ $(x=u(1-v), y=uv\text{ とおく})$

(5) $\frac{p+q}{4}\pi$ $(x=r\cos\theta, y=r\sin\theta\text{ とおく})$

(6) $\left(1-\frac{1}{e}\right)\pi$ $(x=r\cos\theta, y=r\sin\theta\text{ とおく})$

(7) $\frac{8}{15}$ $(x=r\cos\theta, y=r\sin\theta\text{ とおく})$

8. (1) $\frac{2}{3}a^3$ (2) $\frac{4}{3}\pi abc$ (3) $\frac{32}{3}$ (4) $\frac{2}{3}$ (5) $\frac{2}{7}$ (6) $\frac{7\pi}{48}$

(7) 6π (8) $\frac{4}{3}\left(\frac{\pi}{2}-\frac{2}{3}\right)$ (9) $\frac{3}{2}\pi$ (10) $\frac{1}{3}+\frac{\pi}{8}$ (11) $\frac{\pi^2}{16}$

9. (1) $\left(0, \frac{5}{6}, 0\right)$ (2) $\left(0, 0, \frac{3}{8}c\right)$ (3) $\left(\frac{7}{16}, \frac{22}{45}, 0\right)$

10. $\frac{2}{3}\pi\left\{(1+a^2)^{\frac{3}{2}}-1\right\}$

11. $2a^2(\pi+4-4\sqrt{2})$

12. $2\pi a(c-b)$

13. $\frac{12}{5}\pi a^2$