Ghost Busting: Making Sense of Non-Hermitian Hamiltonians

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The Hamiltonian H specifies the energy levels and the time evolution of a quantum theory. It is a conventional axiom of quantum mechanics that H be Hermitian because Hermiticity guarantees that the energy spectrum is real and that the time evolution is unitary (probability preserving). In this talk we investigate an alternative formulation of quantum mechanics in which the usual requirement of Hermiticity (transpose+complex conjugate) is replaced by the more physically transparent condition of space-time reflection (PT) symmetry. We show that if the PT symmetry of a Hamiltonian H is unbroken, then the spectrum of H is real. Examples of PT-symmetric non-Hermitian quantum-mechanical Hamiltonians are $H = p^2 + ix^3$ and $H = p^2 - x^4$. Amazingly, the energy levels of these Hamiltonians are all real and positive!

The crucial question is whether PT-symmetric Hamiltonians specify physically acceptable quantum theories in which the norms of states are positive and the time evolution is unitary. The answer is that a Hamiltonian that has an unbroken PT symmetry also possesses a physical symmetry that we call C. Using C, we show how to construct an inner product whose associated norm *is* positive definite. The result is a new class of fully consistent complex quantum theories for which probabilities are positive and the dynamics is governed by unitary time evolution.

The Lee Model provides an excellent example of a non-Hermitian, PT-symmetric Hamiltonian. The Lee Model was first introduced by T. D. Lee in 1954 as a model quantum field theory in which renormalization can be performed analytically. However, one year later in 1955 it was shown by Kallen and Pauli that the process of renormalizing the Lee Model gives rise to a "ghost" state, which is a strange state of negative norm. This state arises because renormalization causes the Hamiltonian to become non-Hermitian. For the past 50 years there were many attempts to find a physical interpretation of the ghost state, but all these attempts were unsuccessful. I will show that the correct interpretation of the ghost is quite simple: The non-Hermitian Lee Model Hamiltonian is in fact PT-symmetric. I calculate the C operator for the Lee Model exactly and in closed form and show that the ghost state is a completely acceptable physical state having a positive norm.